AN M/M/r QUEUING MODEL FOR BANK TELLER COUNTER IN ABRAKA, DELTA STATE, NIGERIA

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Abstract
Waiting for service is a part of our daily lives. Queues arise when the number of arrivals exceeds the number of servers available. Queuing theory is the mathematical study of waiting lines, which is applicable to the banking system. This study analyzed the queuing situation at First Bank Plc Abraka, putting into consideration the queuing mechanism customers undergo. The First Come First Serve (FCFS) multi-server queuing model was used to follow an exponential distribution. Data used were collected from First Bank Plc Abraka primarily at deposit/withdrawal section of the banking hall for 30days. The data were collected based on the arrival pattern and the service pattern of customers. The data collected were analyzed using Tora Optimization Window Based Software. The result obtained showed that the average arrival rate is 51 customers/hour and the average service rate is 45 customers/hour. Also, from the results, it could be concluded that any addition made on the number of teller counters will help reduce the time customers spent on queue. This study shows that the waiting time of customers can be measured and improved on, likewise reducing queue length of customers increases satisfaction. The analysis of the system shows that the bank needs to increase their teller counters to six(6), because an increase in teller counters will reduce the time customers have to wait in line before being served.

Keywords – Queuing Model, Multiple Servers, Teller Counters, Waiting Line, Bank.

INTRODUCTION

Queuing refers to the process or act of joining a line. When the number of customers requesting service exceeds the number of servers available, queues form (Asogwa *et al*., 2019). Queues are common in places like post offices, bus stops, hospitals, bank counters, and gas stations, to name a few.

The Queuing theory originated from the work of a Danish Engineer (Erlang (1909). Erlang experimented with call congestion and waiting time. After World War II, mathematicians turned to Erlang's work for research (Whittle 2002). Queuing theory is now being used in areas other than the telecommunications industry, such as factories, financial institutions, offices, and hospitals (Mayhew and Smith 2006, Tijms 2003).

Queues form when the number of customers seeking service exceeds the number of services available, or when the inefficient operation of available facilities increases the time required to serve a customer (Kasum *et al*., 2006). Managers can use queuing theory to determine the optional supply of fixed resources needed to meet varying demands (Duckworth 1962). It is also used to calculate capacity needs (Green 2002, Mcmanus *et al*., 2004). Queuing theory is defined as the mathematical study of waiting lines (Onoja *et al*., 2018). Queuing models are particularly useful for predicting system performance measures such as the probability of an empty system, the expected number of customers in the system, the expected number of customers in the queue, and the system's expected waiting time (Cruz *et al*., 2020). It can be applied to a wide range of life situations (Amos 2008). Because of the unpredictable nature of arrival and service patterns and the direct effects of waiting patterns, queues can momentarily overload the system and its subsystems (Tsetimi and Orighoyegha 2021, Tsetimi 2013). The M/M/r queuing model is used in this study. The first M in M/M/r indicates that the arrival times follow a Poisson distribution, the second M indicates exponential service times, and r indicates that there are multiple servers in Kendall's notation. Aderinola *et al*. (2020) stated that the assumptions associated with this type of queuing model are as follows: arrivals are at random at the server stations and follow a First-In, First-Out (FIFO) queue discipline, with no balking or reneging; arrivals follow the Poisson distribution; and the service times follow the negative exponential distribution, hence the service times are also independent, but with a known average.

In this study we shall analyze the queuing situation at deposit/withdrawal point of First Bank Plc, Abraka, Delta State, Nigeria. In order to achieve this, we shall determine the

i. queuing mechanism
ii. parameters of the queuing system
iii. quantitative analysis of the performance measures of the model studied

DESIGN AND SYSTEM DESCRIPTION

The system is modeled as an M/M/r queuing model with relevant data collected from First Bank Plc. The process was observed and primary data were collected for a period of 30days from the 1st day of August to the 9th day of September 2022 excluding Saturdays and Sundays during working hours (9: 00 am – 2: 00 pm) in the bank. The data collected were used to analyze the congestion at the bank with the primary objective of determining the queuing mechanism in the bank, its parameters, and analysis of the performance measures.

The Bank consists of four teller counters for deposit and withdrawal purposes of which one of the teller counters was in the bulk room and three teller counters were then used for this study.

The system has a single queue with three service channels (teller counters). These service channels are parallel to one another and have equal capacity. Each counter can only serve one customer per time. An arriving customer normally joins the already existing queue immediately any teller counter is free, it accepts the next customer on the queue for service. Figure 1 shows a M/M/r model-multi channel single queuing system as observed in the bank.

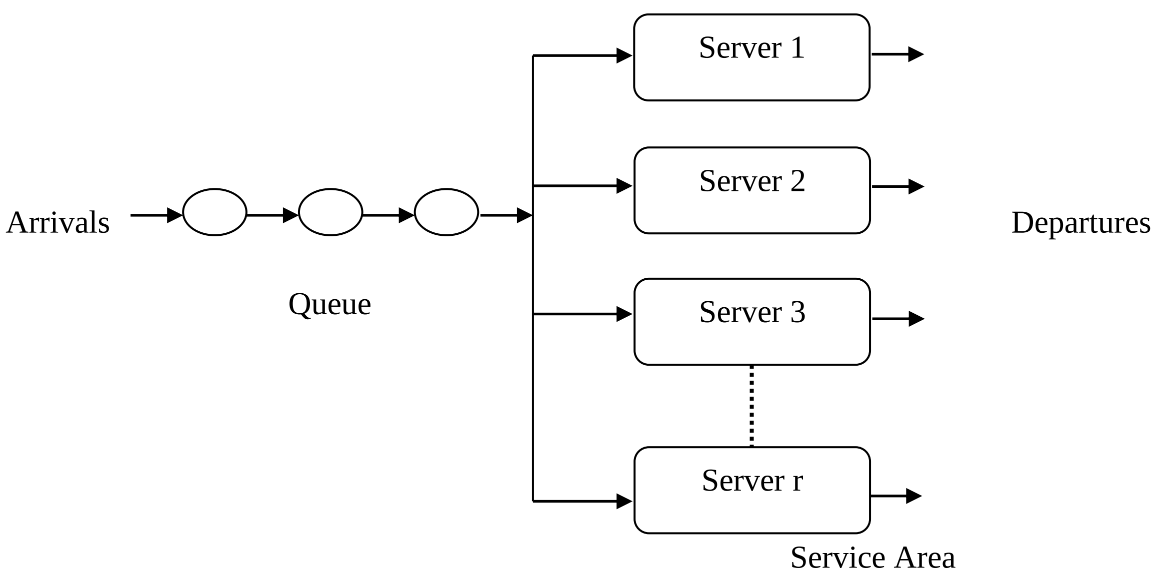

Figure1. Diagrammatic Representation of a M/M/r Model – Multi Channel Model

Since our case study have multiple teller counters (service channels) it implies that multiple servers are attending to customers at any given time. When customers arrive at the bank, they form a single line, waiting in the queue if the system is busy, receive service, and eventually leave.

MODEL SPECIFICATION: M/M/r MODEL

Queuing models are logical descriptions of queuing system behavior. The appropriate queuing model for this study was established as

$$M/M/r/\infty/FCFS$$

The M/M/r model inter arrival and service times are independently and identically distributed (iid) according to an exponential distribution. In this case, the servers (teller counters) are indicated as r. This model is used in a case where r-parallel servers that offer the same service are provided to serve customers from the calling population. If the system has just a single server $(r = 1)$, the parameters for the arrival and departure process are $\lambda_n = \lambda$ and $\mu_n = \mu$, but when the system has multiple servers $(r > 1)$, μ_n denotes the mean service rate for the overall queuing system when the system has n customers. Where μ is the service rate per busy server, the overall mean service rate for n busy servers must then be μ_n, therefore $\mu_n = n\mu$ when $n < r$, and $\mu_n = r\mu$ when $n \geq r$. When the maximum mean service rate $r\mu$ exceeds the mean arrival rate λ, then

$$\rho = \frac{\lambda}{r\mu} \tag{1}$$

The arrival times and service times associated with each server are state independent; however, because the number of servers that actually attend to customers depends on the number of customers in the system, the effective time it takes the system to process customers through the service time facility is state dependent.

The model for this study has the following assumptions:

i. the arrival of customers per unit of time λ into the system follows the Poisson process.
ii. a single waiting line was formed and each arrival waited to be served regardless of the queue length.
iii. the service time μ are exponentially distributed.
iv. the system has multi service channel.
v. the queuing system has an infinite capacity.
vi. the queue discipline is First-Come First-Served (FCFS).
vii. the average arrival rate is greater than the average service rate.

Sharma (2016) summarizes the step-by-step procedure for determining the probability P_n of n customers in a queuing system at time t as follows:

Step1: Obtain the system of differential-difference equations

$$P_n(t+dt) = P_n(t)\{1-\lambda dt\}\{1-n\mu dt\} + P_{n+1}(t)\{1-\lambda dt\}\{(n+1)\mu dt\} + P_{n-1}(t)\{\lambda dt\}\{1-(n-1)\mu dt\} \quad (2)$$

$$= -(\lambda + n\mu)P_n(t)dt + (n+1)\mu P_{n+1}(t)dt + \lambda P_{n-1}(t)dt + P_n(t) + \text{ terms involving}(dt)^2; 1 \geq n < r$$

$$P_n(t+dt) = P_n(t)\{1-\lambda dt\}\{1-\mu dt\} + P_{n+1}(t)\{1-\lambda dt\}\{r\mu dt\} + P_{n-1}(t)\lambda dt\{1-r\mu dt\} \quad (3)$$

$$= -(\lambda + r\mu)P_n(t)dt + r\mu P_{n+1}(t)dt + \lambda P_{n-1}(t)dt + P_n(t) + \text{ terms involving}(dt)^2; 1 \geq r$$

and

$$P_0(t+dt) = P_0(t)(1-\lambda dt) + P_1(t)\mu dt; \quad n = 0 \quad (4)$$

By dividing these equations by dt and then by taking limit as $dt \to 0$, we get

$$\begin{cases} P'_n(t) = -(\lambda + n\mu)P_n(t) + (n+1)\mu P_{n+1}(t) + \lambda P_{n-1}(t) & ; \ 1 \leq n < r \\ P'_n(t) = -(\lambda + r\mu)P_n(t) + r\mu P_{n+1}(t) + \lambda P_{n-1}(t) & ; \ n \geq r \\ P'_0(t) = -\lambda P_0(t) + \mu P_1(t) & ; \ n = 0 \end{cases} \quad (5)$$

Step 2: Obtain the system of steady-state equations

In the steady state condition, the differential-difference equations obtained from equation (5)as $t \to \infty$, are:

$$-\lambda P_0 + \mu P_1 = 0 \quad ; \quad n = 0$$

$$-(\lambda n\mu)P_n + (n+1)\mu P_{n+1} + \lambda P_{n-1} = 0 \quad ; \quad 0 < n < r$$

$$-(\lambda + r\mu)P_n + r\mu P_{n+1} + \lambda P_{n-1} = 0 \quad ; \quad n \geq r$$

Step 3: Solve the system of difference equations

Applying the iterative method, the probability of n customers in the system is given by:

$$P_n = \begin{cases} \dfrac{\rho^n}{n!} P_0 & ; \quad n \le r \\ \dfrac{\rho^n}{r!\, r^{n-r}} P_0 & ; \quad n > r; \quad \rho = \lambda/_{r\mu} \end{cases} \tag{6}$$

Using the following condition, we can find the value of P_0 as:

$$1 = \sum_{n=0}^{\infty} P_n = \sum_{n=0}^{r-1} P_n + \sum_{n=r}^{\infty} P_n = \sum_{n=0}^{r-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0 + \sum_{n=r}^{\infty} \frac{1}{r!\, r^{n-r}}\left(\frac{\lambda}{r\mu}\right)^n P_0$$

$$1 = P_0\left[\sum_{n=0}^{r-1} \frac{r^n}{n!}\left(\frac{\lambda}{r\mu}\right)^n + \sum_{n=r}^{\infty} \frac{r^n}{r!\, r^{n-r}}\left(\frac{\lambda}{r\mu}\right)^n P_0\right] \tag{7}$$

$$= P_0\left[\sum_{n=0}^{r-1} \frac{(r\rho)^n}{n!} + \frac{r^r}{r!}\sum_{n=r}^{\infty} \rho^n\right] = P_0\left[\sum_{n=0}^{r-1} \frac{(r\rho)^n}{n!} + \frac{r^r}{r!}\frac{\rho^r}{1-\rho}\right]; \rho = \frac{\lambda}{r\mu}$$

$$\left[\text{Since} \sum_{n=r}^{\infty} \rho^n = \rho^r + \rho^{r+1} + \cdots = \frac{\rho^r}{(1-\rho)}, \text{sum of infinite G.P.}; \rho < 1\right]$$

Hence the probability that the system is idle is:

$$P_0 = \left[\sum_{n=0}^{r-1} \frac{(r\rho)^n}{n!} + \frac{1}{r!}\frac{(r\rho)^r}{1-\rho}\right]^{-1} \quad ; \quad \rho = \frac{\lambda}{r\mu}$$

$$= \left[\sum_{n=0}^{r-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{r!}\left(\frac{\lambda}{\mu}\right)^r \frac{r\mu}{r\mu - \lambda}\right]^{-1} \tag{8}$$

where

$r =$ Number of service channels

$\lambda =$ Average arrival rate

$\mu =$ Average service rate

$\rho =$ Utilization rate

$P_0 =$ probability of having no customer in the system

Other parameters are

Probability that a customer has time to wait for service:

$$P(n \geq r) = \frac{1}{r!}\left(\frac{\lambda}{\mu}\right)^r \left(\frac{r\mu}{r\mu - \lambda}\right)\rho_o \quad (9)$$

Average number of customers in the waiting line:

$$L_q = \left[\frac{1}{(r-1)!}\left(\frac{\lambda}{\mu}\right)^r \frac{\lambda\mu}{(r\mu - \lambda)^2}\right]\rho_o \quad (10)$$

Average number of customers in the system

$$L_s = L_q + \rho$$

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$L_s = \left[\frac{1}{(r-1)!}\left(\frac{\lambda}{\mu}\right)^r \frac{\lambda\mu}{(r\mu - \lambda)^2}\right]\rho_o + \frac{\lambda}{\mu} \quad (11)$$

Average time a customer spends in the waiting line

$$W_q = \frac{\left[\frac{1}{(r-1)!}\left(\frac{\lambda}{\mu}\right)^r \frac{\lambda\mu}{(\lambda\mu - \lambda)^2}\right]\rho_o}{\lambda} \quad (12)$$

Average time a customer spends in the system

$$W_s = W_q + \frac{1}{\mu}$$

$$W_s = \frac{\left[\frac{1}{(r-1)!}\left(\frac{\lambda}{\mu}\right)^r \frac{\lambda\mu}{(\lambda\mu - \lambda)^2}\right]\rho_o}{\lambda} + \frac{1}{\mu} \quad (13)$$

Traffic Intensity and Measures of Performance

The following are the formulae for measuring traffic intensity and performance at First Bank PLC, Abraka.

i. Probability of having no customer in the system (P_0)

$$\rho_o = \left[\sum_{n=0}^{r-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{r!}\left(\frac{\lambda}{\mu}\right)^r \cdot \frac{r\mu}{(r\mu - \lambda)}\right]^{-1} \qquad (14)$$

$\rho = \frac{\lambda}{r\mu}$ =utilization rate, $for\ n \geq 3$

r =number of service channels

λ =average arrival rate

μ =average service rate

ii. The expected number of customers in the queue (L_q)

$$L_q = \left[\frac{1}{(r-1)!}\left(\frac{\lambda}{\mu}\right)^r \frac{\lambda\mu}{(r\mu - \lambda)^2}\right]\rho_o \qquad (15)$$

iii. The expected number of customers in the system (L_s)

$$L_s = L_q + \frac{\lambda}{\mu} \qquad (16)$$

iv. The average waiting time in the queue (W_q)

$$W_q = \frac{L_q}{\lambda} \qquad (17)$$

v. The average waiting time in the system (W_s)

$$W_s = W_q + \frac{1}{\mu} \qquad (18)$$

vi. The probability that a customer has to wait $(P(n \geq r))$

$$P(n \geq r) = \frac{1}{r!}\left(\frac{\lambda}{\mu}\right)^r \left(\frac{r\mu}{r\mu - \lambda}\right)\rho_o \qquad (19)$$

Equations 1 – 13are used to calculate the relevant parameters for our models. We also applied Equations 14 – 19 for the traffic intensity and measures of performance. The calculated values are presented in Tables 2 -4. Table 1 shows the observed arrival and service times of customers at First Bank Abraka.

Table 1: Summary and presentation of analyzed data for customers that arrived and were served at First Bank Plc Abraka, Delta State 2022

Days	Date	Time (Hours)	Number of customers arrival	Number of customers served
Day 1	1/8/2022	5	433	415
Day 2	2/8/2022	5	131	114
Day 3	3/8/2022	5	149	126
Day 4	4/8/2022	5	219	194
Day 5	5/8/2022	5	384	370
Day 6	8/8/2022	5	304	295
Day 7	9/8/2022	5	108	88
Day 8	10/8/2022	5	235	211
Day 9	11/8/2022	5	140	120
Day 10	12/8/2022	5	410	380
Day 11	15/8/2022	5	425	404
Day 12	16/8/2022	5	125	115
Day 13	17/8/2022	5	186	154
Day 14	18/8/2022	5	142	128
Day 15	19/8/2022	5	364	343
Day 16	22/8/2022	5	402	382
Day 17	23/8/2022	5	207	179
Day 18	24/8/2022	5	225	174
Day 19	25/8/2022	5	220	192
Day 20	26/8/2022	5	345	324
Day 21	29/8/2022	5	382	373
Day 22	30/8/2022	5	106	86
Day 23	31/8/2022	5	245	220
Day 24	1/9/2022	5	233	211
Day 25	2/9/2022	5	415	387
Day 26	5/9/2022	5	381	360
Day 27	6/9/2022	5	113	97
Day 28	7/9/2022	5	158	130
Day 29	8/9/2022	5	113	95
Day 30	9/9/2022	5	350	339
Total		150	7650	6760

Source: Field data (2022)

Presentation of Analyzed Data

Table 2 indicates the Day, Time, Average arrival rate(λ), Average departure rate(μ) in First Bank Plc Abraka.

Table 2: Average of the summary of data and daily queuing system analysis of the teller counter-based number of hours of data collection 5 hours

Days	Time	λ	μ
1	9:00a.m – 2:00p.m	87	80
2	9:00a.m – 2:00p.m	26	23
3	9:00a.m – 2:00p.m	30	25
4	9:00a.m – 2:00p.m	44	35
5	9:00a.m – 2:00p.m	77	72
6	9:00a.m – 2:00p.m	61	55
7	9:00a.m – 2:00p.m	22	18
8	9:00a.m – 2:00p.m	47	42
9	9:00a.m – 2:00p.m	28	24
10	9:00a.m – 2:00p.m	82	76
11	9:00a.m – 2:00p.m	85	74
12	9:00a.m – 2:00p.m	25	23
13	9:00a.m – 2:00p.m	37	31
14	9:00a.m – 2:00p.m	28	26
15	9:00a.m – 2:00p.m	73	62
16	9:00a.m – 2:00p.m	80	64
17	9:00a.m – 2:00p.m	41	36
18	9:00a.m – 2:00p.m	45	35
19	9:00a.m – 2:00p.m	44	37
20	9:00a.m – 2:00p.m	69	65
21	9:00a.m – 2:00p.m	76	73
22	9:00a.m – 2:00p.m	21	16
23	9:00a.m – 2:00p.m	49	44
24	9:00a.m – 2:00p.m	47	41
25	9:00a.m – 2:00p.m	83	76
26	9:00a.m – 2:00p.m	76	72
27	9:00a.m – 2:00p.m	23	20
28	9:00a.m – 2:00p.m	32	26
29	9:00a.m – 2:00p.m	23	17
30	9:00a.m – 2:00p.m	70	66
Total	1531	1531	1354

Source: Field data (2022)

Table 3 shows the Day, Average arrival rate(λ), Average departure rate(μ), Traffic intensity (ρ), Probability that the teller counters are idle(P_0), Average number of customers in the system (L_s), Average number of customers in the queue(L_q) , Average time spent in the system(W_s) and Average time spent in the queue (W_q) for each day in First Bank Plc Abraka.

Table 3: Performance measure analysis for First Bank Plc Abraka, Delta State using M/M/3 queuing model

Days	λ	μ	ρ	P_0	L_s	L_q	W_s	W_q
1	87	80	0.36250	0.33167	1.15091	0.06341	0.01323	0.00073
2	26	23	0.37681	0.31689	1.20446	0.07402	0.04633	0.00285
3	30	25	0.40000	0.29412	1.29412	0.09412	0.04314	0.00314
4	44	35	0.41905	0.27645	1.37080	0.11366	0.03115	0.00258
5	77	72	0.35648	0.33806	1.12877	0.05932	0.01466	0.00077
6	61	55	0.36970	0.32416	1.17768	0.06859	0.01931	0.00112
7	22	18	0.40741	0.28714	1.32359	0.10137	0.06016	0.00461
8	47	42	0.37302	0.32075	1.19013	0.07108	0.02532	0.00151
9	28	24	0.38889	0.30485	1.25068	0.08402	0.04467	0.00300
10	82	76	0.35965	0.33468	1.14040	0.06145	0.01391	0.00075
11	85	74	0.38288	0.31079	1.22757	0.07892	0.01444	0.00093
12	25	23	0.36232	0.33186	1.15024	0.06329	0.04601	0.00253
13	37	31	0.39785	0.29617	1.28564	0.09209	0.03475	0.00249
14	28	26	0.35897	0.33540	1.13792	0.06099	0.04064	0.00218
15	73	62	0.39247	0.30135	1.26460	0.08718	0.01732	0.00119
16	80	64	0.41667	0.27861	1.36105	0.11105	0.01701	0.00139
17	41	36	0.37936	0.31404	1.21516	0.07627	0.02964	0.00186
18	45	35	0.42857	0.26794	1.41029	0.12457	0.03134	0.00277
19	44	37	0.39640	0.29756	1.27933	0.09074	0.02909	0.00206
20	69	65	0.35385	0.34089	1.11914	0.05760	0.01622	0.00083
21	76	73	0.34703	0.34830	1.09441	0.05332	0.01440	0.00070
22	21	16	0.43750	0.26016	1.44806	0.13556	0.06896	0.00646
23	49	44	0.37121	0.32260	1.18336	0.06972	0.02415	0.00142
24	47	41	0.38211	0.31155	1.22463	0.07829	0.02606	0.00167
25	83	76	0.36404	0.33005	1.15660	0.06449	0.01393	0.00078
26	76	72	0.35185	0.34304	1.11187	0.05632	0.01463	0.00074
27	23	20	0.48333	0.31034	1.22930	0.07930	0.05345	0.00345
28	32	26	0.41026	0.28449	1.33504	0.10427	0.04172	0.00326
29	23	17	0.45098	0.24876	1.50656	0.15362	0.06550	0.00668
30	70	66	0.35354	0.34122	1.11800	0.05740	0.01597	0.00082

Source: Field data (2022)

Based on available data, the parameters calculated are as follows:

Average customer arrival rate, $\lambda = \frac{7650}{150} =$ 51 customers/hour

Average customer service rate, $\mu = \frac{6760}{150} =$ 45 customers/hour

Table 3 shows the performance measures of the bank with increase in the number of teller counters, ranging from Average arrival rate (λ), Average departure rate (μ), Traffic intensity (ρ), Probability that the teller counters are idle (P_0), Probability that a customer has to wait ($P(n \geq r)$), Average number of customers in the system (L_s), Average number of customers in the queue (L_q), Average time spent in the system (W_s) and Average time spent in the queue (W_q) for each day in First Bank Plc Abraka. While Figure 2 shows the peak periods in the bank.

Table 4: Summary of the performance measures of the bank with different number of teller counters

r	3	4	5	6
λ	51	51	51	51
μ	45	45	45	45
ρ	0.37778	0.28333	0.22667	0.18889
P_0	0.31591	0.32112	0.32185	0.32194
$P(n \geq r)$	0.12318	0.03080	0.00648	0.00117
L_s	1.20812	1.14551	1.13523	1.13661
L_q	0.07479	0.01218	0.00190	0.00027
$W_s(hours)$	0.02369	0.02246	0.02226	0.02223
$W_q(hours)$	0.00147	0.00024	0.00004	0.00001

Source: Field data (2022)

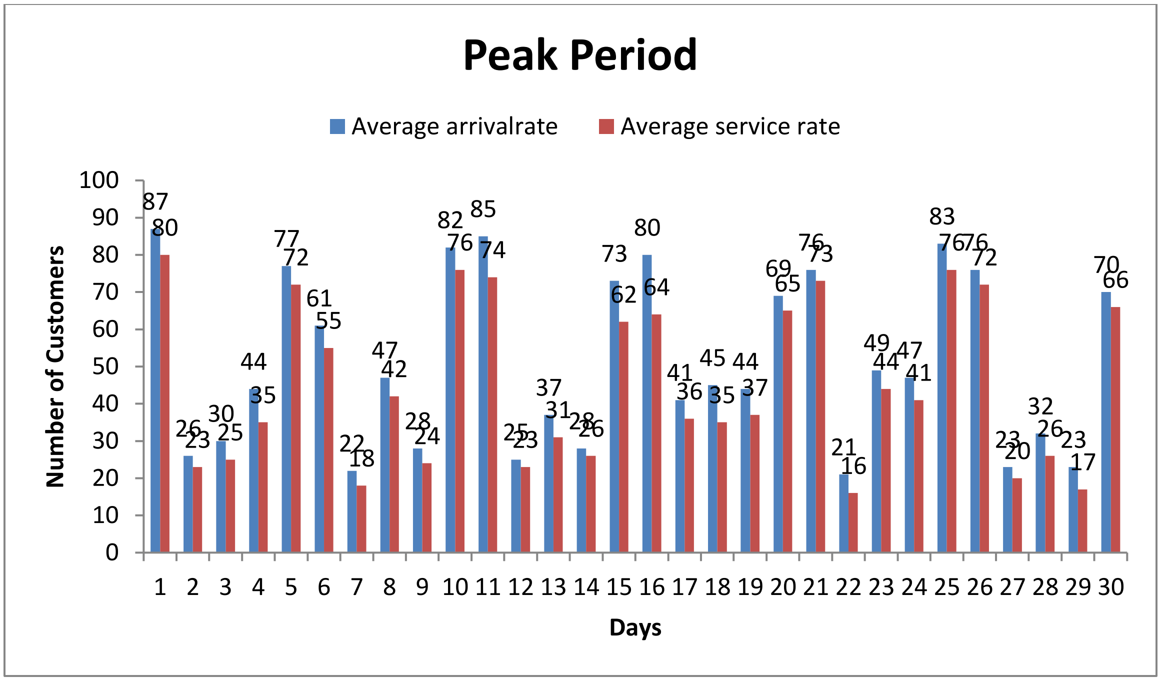

Figure 2: Peak period in First Bank Plc Abraka, Delta State, Nigeria

Discussion

From the quantitative analysis, the capacity of the system under study is 7650customers and the average arrival rate is 51 customers/hour while the average service rate is 45 customers/hour. The arrival rate being greater than the service rate implies that customers have to queue up. The probability that the teller counters are idle is 0.31591 which shows that 32% idle and 68% busy. It can be seen from Figure 2 that the system was busiest on the 1st, 10th, 11th and 25th day and during each week the busiest days are Mondays and Fridays.

From Table 4, it can be seen that when the number of service channel (r) is 3, the utilization rate $(\rho) = 0.37778$. Furthermore, the probability that a customer has to wait $(P(n \geq r)) = 0.12318$. The probability of having no customer in the system$(P_0) = 0.31591$, the probability that a customer enters the system without waiting $P(n < r) = 0.74510$, expected number of customers in the system $(L_s) = 1.20812$, expected number of customers on queue $(L_q) = 0.07479$, waiting time in the system $(W_s) = 0.02369$ hours, and waiting time in the queue $(W_q) = 0.00147$hours. Also, from Table 4, the increment in the number of teller counters from three (3) to four (4) show a decrease in the waiting time of customers in the queue (W_q) from 0.00147 hours (0.0882 minutes) to 0.00024 hours (0.0144 minutes), while the waiting time of customers in the system (W_s) decreases from 0.02369 hours (1.4214 minutes) to 0.02246 hours (1.3476 minutes). Increasing the servers from four (4) to five (5), show a decrease in the waiting time of customers in the queue (W_q) from 0.00024 hours (0.0144 minutes) to 0.00004 hours (0.0024 minutes), while the waiting time of customers in the system (W_s) decreases from 0.02246 hours (1.3476 minutes) to 0.02226 hours (1.3356 minutes). By further increment, from five (5) to six (6), the waiting time of customers in the queue (W_q) decreases from 0.00004 hours (0.0024 minutes) to 0.00001 hours (0.0006 minutes), and the waiting time of customers in the system (W_s) decreases from 0.02226 hours (1.3356 minutes) to 0.02223 hours (1.3338 minutes).

Conclusion

From the discussion of results, it could be concluded that any addition made on the number of teller counters will help reduce the time customers spend on queue.

The queuing model used for this study is M/M/3 at First Bank Plc Abraka with analysis for when teller counters (service channels) are increased to 4,5, and 6.The traffic intensity at First Bank Plc Abraka for M/M/3 was found to be 38%. At the teller counter of First Bank, the parameters derived were the utilization factor ρ, average time spent on the queue (W_q), average time spent in the system (W_s) likewise average number of customers on the queue (L_q), average number of customers in the system (L_s). Other parameters are the probability that the system is empty P_0 and probability that a customer has to wait $P(n \geq r)$.This study was able to establish the queuing mechanism of the teller counters at First Bank Plc Abraka. It was able to obtain the peak periods in the bank and give appropriate feedback of performance to help improve the system. It proffered ways to help the Bank management make decisions on how to optimize the overall systems with queuing models when incorporated to help improve the system.

References

Aderinola, O. S., Elemure, V. A. & Laoye, A. A. (2020). Analysis of traffic congestion at Jattu Junction (Auchi) with queuing theory using TORA and SIDRA softwares. *Global*

Journal of Engineering and Technology Advances, *3*(1), 010-014.

Amos, H. (2008). Simulation of Restaurant Operations using the Restaurant Modeling Studies. In *Proceedings of the 2002 Winter Simulation Conference.* IEEE Press.(2002)144-1453.

Asogwa, O. C., Eze, C. M., & Edeaja, M. T. (2019). On the Application of Queuing Model in Nigeria Banking Sector: A Case Study of Zenith Bank Plc, Abakaliki Branch. *Global Journal of Science Frontier Research: Mathematics and Decision Science*, *19*(2), 51-58.

Cruz, F. R., Quinino, R. C., & Ho, L. L. (2020). Control Charts for Traffic Intensity Monitoring of Markovian Multiserver Queues. *Quality and Reliability Engineering International*, *36*(1), 354-364.

Duckworth, W. E. (1962). Operational research techniques. *Methinen and co., London*.

Erlang, A. K. (1909). The Theory of Probabilities and Telephone Conversations. *Nyt Tidsskrift for Matematik B*, *20*, 33-39.

Green, L. V. (2002). How many hospital beds?. *INQUIRY: The Journal of Health Care Organization, Provision, and Financing*, *39*(4), 400-412.

Kasum, A. S., Abdulraheem, A., & Olaniyi, T. A. (2006). Queue Efficiency in Nigeria Banks: A Comparative Analysis of Old and New Generation Banks. *Ilorin Journal of Sociology*, *2*(1), 162-172.

Mayhew, L., & Smith, D. (2006). Using Queuing Theory to Analyze Completion Times in Accident and Emergency Departments in the Light of the Government 4-Hour Target.

McManus, M. L., Long, M. C., Cooper, A., & Litvak, E. (2004). Queuing theory accurately models the need for critical care resources. *The Journal of the American Society of Anesthesiologists*, *100*(5), 1271-1276.

Onoja, A. A., Babasola, T., Moyo, E., & Ojiambo, V. (2018).The Application of Queuing Analysis in Modeling Optimal Service Level. *International Journal of Scientific & Engineering Research*, *9*(1), 184-194.

Sharma, J. K. (2016). *Operations Research: Theory and Applications* (Sixth). Trinity Press an imprint of Laxmi Publications.

Tijms, H.C. (2003). Algorithmic Analysis of Queues: A First Course in Stochastic Models. Chichester: Wiley.

Tsetimi, J. (2013), A survey of some Queuing Models in Muti-stage Manufacturing system, *The Journal of Nigerian Institution of Production Engineers*. (2013) 15:195-201.

Tsetimi, J. & Orighoyegha, M.G. (2021).An M/M/S Queuing Model for Selected Tank Farms in Oghara, Delta State, Nigeria. *International Journal of Mathematics Trends and Technology*. *67 (2)* 29-35.

doi:10.14445/22315373/JMTTV67I2P505

Whittle, P. (2002). Applied Probability in Great Britain. *Operations Research*, *50*(1), 227-239.