AN EXPLICIT ANALYSIS OF THE COVARIANCE SYSTEM IN A COVARIANCE ASSIGNMENT PROBLEM

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Abstract

System analysis is essential in control theory. Stability, controllability and observability are vital issues to be considered in control systems .Different methods were used for the stability of linear systems. One of these methods is the Jury's stability criterion used to ascertain the stability of discrete-time systems .This study investigates the stability of the corresponding covariance system of a Discrete-Time Linear Time-Invariant Stochastic Dynamical System (DTLTISDS) in a covariance assignment problem (CAP) via the Jury's stability criterion. The characteristics equation was obtained from the transfer function of the covariance system. Necessary and sufficient conditions for stability was investigated utilizing the constant coefficients of resulting polynomial with respect to the characteristics equation and the Jury's table .The Jury's table was constructed with the aid of the constant coefficients of the polynomial and the Jury's inner determinant methods. Kalman's rank test was used to analyze controllability and observability. Results show that the covariance system is a stable system. The covariance system was also shown to be controllable and observable.

Keywords: covariance system, covariance assignment, stability, controllability, observability

1. INTRODUCTION

Covariance assignment problems have been prominent in literature for the past three decades. Covariance assignment is a typical problem in control system which involves the design of efficient controllers so as to assign a covariance value to the state of the system (Baromand and Khaloozadeh, 2007). The idea of covariance control was first introduced by Hotz and Skelton 1987. Since the performance requirement of most engineering systems are stated in terms of the variance of the system's states, the need for covariance control became necessary. This is also due to the fact that the various theories of identification, estimation and model reduction use covariance as a measure of performance. Covariance assignment for Discrete Multiple-Input-Multiple-Output (MIMO) systems has been addressed (Skelton and Collins, 1987). An extension of this study was presented for Continuous Multiple-Input-Multiple-Output (MIMO) systems and dynamic controllers (Hotz and Skelton, 1987). The covariance assignment problem was also extended with the development of minimum energy covariance controllers (Grigoriadis and Skelton, 1993; Grigoriadis and Skelton, 1997). During the late 90's, Fujioka and Hara also made notable contributions in the area of covariance assignment (Fujioka and Hara, 1994; Fujioka and Hara, 1995).

The covariance system is a usually a linear deterministic system, although the original dynamical system might be a stochastic linear or nonlinear system (Baroumand, Zaman and Mahmoudi, 2020). The standard state-space model of the covariance system is deduced from a dynamic matrix Riccatti equation defined as (Baromand, Khaloozadeh and Mohamadreza, 2007):

$$P(k+1) = AP(k)A^T + BU(k)B^T + DQ(k)D^T. \qquad (1)$$

Dynamical systems change trajectories (i.e. acceleration, velocity, position). They are systems that are not static due to the fact that their states evolve with respect to time as a result of input signals, external perturbations or natural causes (Ducard, 2017).The state of a dynamical system can be represented as a state vector. If the dynamical system is a linear time-invariant finite-dimensional system, then the differential and algebraic expressions can be written in matrix form (Hangos et al, 2004). Deterministic dynamical systems are predictable systems, since their state changes over time according to a rule. On the other hand stochastic dynamical system also referred to as chaotic dynamical systems are unpredictable. Stochastic dynamical systems which are usually subjected to randomness and a lot of uncertainties have become a major research area due to the fact that, they are often present in real life systems (Serborg et al, 2017).

Systems analysis is very essential in all control systems. These analyses are usually investigated based on different criteria with respect to stability, controllability, observability, realizability and so on. To be more specific, the issue of stability in control systems has been addressed by several authors including Jury (Jury, 1961; Jury, 1965). Stability is the most important characteristics possessed by all types of systems (Ramesh and Manikanda, 2015). In the case of Linear Time Invariant Discrete system which is represented by its characteristics equation $f(z) = 0$, the system is called a stable system if $|z| < 1$.

Different definitions, methods and different algebraic scheme for stability have been presented in the literature. These criteria includes the Lyapunov stability criteria, transfer function, the Routh Hurwitz criterion, Jury's criterion, impulse function, long division method, the Popov criterion etc (Piriadarshani and Sujitha (2018); Matousek et al. (2009); Fahri and Metin, 2019). The main goal of analyzing systems is to actually gain a better understanding of the behavior of the solution to the system. The natural approach to analyze a system is to solve it explicitly. This method is very efficient for linear systems of lower order. However for higher order systems it becomes very difficult to analyze stability manually.

Stability of Discrete-time linear time-invariant systems is determined by the root location of the systems characteristics polynomial with respect to the unit circle. The system is stable if and only if all roots lie in the unit circle. Jury, presented a method to ascertain if the roots of a system characteristic polynomial lie inside or outside the unit circle in the z-domain (Seyed and Amir, 2012). The characteristics equation of a discrete-time system is given as (Fahri and Metin, 2019):

$$q(z) = a_n z^n + a_{n-1} z^{n-1} + a_{n-2} z^{n-2} + \cdots + a_2 z^2 + a_1 z + a_0 = 0, \qquad (2)$$

Where a_i's are the coefficients and n is the degree of $(z) = 0$.

If the coefficients of z^n is not unit we can divide all the coefficients by a_n. Thus equation (2) can be written as (Ramesh and Manikandan, 2014):

$$q(z) = z^n + \left(\frac{a_{n-1}}{a_n}\right) z^{n-1} + \cdots\cdots\cdots + \left(\frac{a_0}{a_n}\right) = 0 \, . \qquad (3)$$

Discrete-time systems can be represented either as a difference equation or a transfer function:

$$H(z) = \frac{Y(z)}{X(z)} \quad , \qquad (4)$$

Where $Y(Z)$ and $X(Z)$ represents a transform of the output and a transform of the input respectively.

Controllability and observability are two fundamental concepts in mathematical control theory. Studies on the controllability and observability for the linear case started in the early 1960s (Kalman, 1960).This study led to the development of the concept of observability of a linear time-invariant system (Whalen et al, 2015). Analysis of these dual concepts was extended to nonlinear systems in 1970s (Hermann and Krener, 1977). Observability and controllability analysis has been subjected to much research in the literature (Zhirabok and Shumsky, 2012; Ge, 2021; Erfan, 2020; Wu et al, 2020; Stigter et al, 2018; Naim et al, 2018).

Zhang et al. (2018) presented useful theories about the covariance control of perturbed bilinear or nonlinear stochastic system. In order to motivate a better understanding of the state covariance assignment idea of a linear system, theories on how to determine the desired set of state covariance of nonlinear systems, multivariable system and multisensory systems have also been investigated (Kalandros, 2002; Baromand and Labibi, 2012).The finite-Horizon covariance control problem has been addressed by (Bakolas, 2018; Goldshtein, and Tsiotras, 2017; Halder and Wendel, 2016). An optimal control problem for stochastic discrete-time systems has been presented (Okamato and Tsiotras, 2019; Bakolas, 2016). Recently, Baroumand et al. (2020) studied the covariance control algorithm for non-linear stochastic systems using covariance control method and linear approximations of non-linear systems. Zare et al. (2016) presented a study based on the idea of covariance completion in linear dynamical systems. . Numerous studies have been presented in existing literature on the covariance assignment problem, stabilization of dynamical systems in covariance assignment problems (CAP) has been achieved by using appropriate controllers. However, an explicit stability analysis of the covariance system in a covariance assignment problem (CAP), via the Jury's criterion has not been known to us in the literature. Thus we intend to analyze the stability of the corresponding covariance system of a Discrete-Time Linear Time-Invariant Stochastic Dynamical System (DTLTISDS) in a Covariance Assignment Problem (CAP) via the Jury's criterion, as well as to also analyze controllability and observability of the covariance system using Kalman's rank test.

2. MATERIALS AND METHOD

In this study, the corresponding covariance system of a DTLTISDS in a Covariance Assignment Problem (Baromand et al, 2007) is considered and investigated.

Consider a state-space representation of a linear discrete-time stochastic system in a covariance assignment problem given as (Baromand et al., 2007):

$$x(k+1) = Ax(k) + Bu(k) + Dw(k) \ , \quad (5a)$$

$$y(k) = Cx(k), \quad (5b)$$

Where $x(k) \in R^n, u(k) \in R^m, w(k) \in R^l \ and \ y(k) \in R^P$ represents the states,input control signal , white process noise and the output respectively. $A, B, C, D$are matrices in $R^{n\times n}, R^{n\times m}, R^{p\times m}, R^{n\times l}$respectively.

It is assumed that the white noise vector satisfies:

$$\left.\begin{array}{c} E[w(k)] = 0, \\ E[x(0)w^T(k)] = 0, \\ E[x(0)w^T(j)] = M\delta(i-j). \end{array}\right\} \quad (6)$$

$$\text{where,} \delta(i-J) = \begin{cases} 1, if i = j \\ 0, if i \neq j \end{cases}$$

Note that $M \in R^{l\times l}$ is the covariance matrix of $w(t)$.

The control signal satisfies:

$$\left.\begin{array}{c} E[u(k)]0, \\ E[x(0)u^T(K)] = 0, \\ E[u(k_1)w^T(k_2)] = 0, \forall \ k_1, k_2 \\ E[u(i)u^T(J)] = U(i)\delta(i-j). \end{array}\right\} \quad (7)$$

$$\text{where, } \delta(i-j) = \begin{cases} 1, if \ i = j \\ 0, if \ i \neq j \end{cases}$$

2.1 Covariance System Description in the CAP

The dynamics of the covariance system is determined by the equation (1),where $P(k), U(k) \ and \ Q(k)$ denotes the state, input and noise covariance matrices respectively (Baromand et al, 2020).

The system state vectors $x(k) = [x_1, x_2, \cdots, x_n]^T$,

The system input vectors $u(k) = [u_1, u_2, \cdots, u_m]^T$, The system noise vectors $w(k) = [e_1, e_2, \cdots, e_l]^T$.

Thus,

$$P(k) = E[x(k)x^T(k)], \tag{8}$$

$$U(k) = E[u(k)u^T(k)], \tag{9}$$

$$Q(k) = E[w(k)w^T(k)]. \tag{10}$$

Equation (8), (9) and (10) can be partitioned as a $\frac{n\times(n+1)}{2}\times 1, \frac{m\times(m+1)}{2}\times 1$ and $\frac{l\times(l+1)}{2}$ vector to obtain $P_{cov}(k), U_{cov}(k)$ and $Q_{cov}(k)$ respectively.

The system (1) can be reduced to the standard state space model of the covariance system given as (Khaloozadeh and Baromand, 2010):

$$P_{cov}(k+1) = A_{cov}P_{cov}(k) + B_{cov}U_{cov}(k) + D_{cov}Q_{cov}(k), \tag{11}$$

where,

$A_{cov} \in R^{n(n+1)/2\times n(n+1)/2}$, $B_{cov} \in B^{n(n+1)/2\times m(m+1)/2}$, $D_{cov} \in R^{n(n+1)/2\times l(l+1)/2}$ represents matrices of the state, the input and disturbance matrices of the covariance system respectively.

$D_{cov}Q_{cov}$ can be assumed to be a disturbance term because it is an uncontrollable term. So that we can rewrite equation (13) as:

$$P_{cov}(k+1) = A_{cov}P_{cov}(k) + B_{cov}U_{cov}(k) + d\ . \tag{12}$$

The controllability of the pair $(A_{cov}B_{cov})$ is a necessary condition for the existence of a solution in a covariance assignment problem.

Consider the system (5a) and (5b) with matrices A, B, D .Let,

$$A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \qquad B = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} \qquad D = \begin{bmatrix} d_1 & d_2 \\ d_3 & d_4 \end{bmatrix}$$

The elements of A_{cov} ,B_{cov}, $and\ D_{cov}$ can be obtained using (Baromand 2007):

$$A_{cov} = \begin{bmatrix} {a_1}^2 & 2a_1a_2 & {a_2}^2 \\ a_1a_3 & a_2a_3 + a_1a_4 & a_4a_2 \\ {a_3}^2 & 2a_3a_4 & {a_4}^2 \end{bmatrix},\ B_{cov} = \begin{bmatrix} {b_1}^2 & 2b_1b_2 & {b_2}^2 \\ b_1b_3 & b_2b_3 + b_1b_4 & b_4b_2 \\ {b_3}^2 & 2b_3b_4 & {b_4}^2 \end{bmatrix},$$

$$D_{cov} = \begin{bmatrix} {d_1}^2 & 2d_1d_2 & {d_2}^2 \\ d_1d_3 & d_2d_3 + d_1d_4 & d_4d_2 \\ {d_3}^2 & 2d_3d_4 & {d_4}^2 \end{bmatrix}.$$

$P_{cov}(k)$, $U_{cov}(k)$ *and* $Q_{cov}(k)$ can also be obtained as follows:

$$P_{\ cov}(k) = \begin{bmatrix} p_1(k) \\ p_2(k) \\ p_3(k) \end{bmatrix} = \begin{bmatrix} var(x_1(k)) \\ cov(x_1(k), x_2(k)) \\ var(x_2(k)) \end{bmatrix},$$

$$U_{cov}(k) = \begin{bmatrix} var(u_1(k) \\ cov(u_1(k), u_2(k)) \\ var(u_2(k)) \end{bmatrix},\ Q_{cov}(k) = \begin{bmatrix} var(e_1(k)) \\ cov(e_1, e_2) \\ var(e_2(k)) \end{bmatrix}.$$

2.2 Stability analysis of the covariance system in the CAP

The stability of the corresponding covariance system of the Discrete-Time Linear Time-invariant stochastic system (DTLTISS) in the CAP is addressed in the 'z' domain using the jury's criterion. Consider the linear discrete-time stochastic system (5a) and (5b) in a covariance assignment problem (Baromand et al 2007) :

Given the matrices $= \begin{bmatrix} -0.5 & -1.3 \\ 0.1 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0.5 \\ 0.1 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}$

The corresponding state, input and output matrices of the covariance system are obtained as:

$$A_{cov} = \begin{bmatrix} 0.25 & 1.3 & 1.69 \\ -0.05 & -0.03 & 0.26 \\ 0.01 & -0.04 & 0.04 \end{bmatrix}, \quad B_{cov} = \begin{bmatrix} 4 & 1.2 & 0.09 \\ 0.2 & 8.03 & 1.2 \\ 0.01 & 0.8 & 16 \end{bmatrix}, \quad C_{cov} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Where $P_{cov}(k) = \begin{bmatrix} var(x_1) \\ cov(x_1, x_2) \\ var(x_2) \end{bmatrix}$

The input-output transfer function $\bar{\bar{G}}$ of the state covariance system is obtained as:

$$\frac{Y_1}{U_1} = \bar{\bar{G}}_{11}(z) = \frac{4z^2+0.2369z+1.6767\times10^{-2}}{z^3-0.26z^2+5.98\times10^{-2}z-1.2167\times10^{-2}} \quad , \tag{13a}$$

$$\frac{Y_2}{U_2} = \bar{\bar{G}}_{22}(z) = \frac{16z^2+3.5671z+0.947807}{z^3-0.26z^2+5.98\times10^{-2}z-1.2167\times10^{-2}} \quad . \tag{13b}$$

The denominator of the (13a) and (13b) is the characteristics equation of the covariance system with respect to the Discrete-time linear Time-invariant stochastic system given as:

$$z^3 - 0.26z^2 + 5.98 \times 10^{-2} - 1.2167 \times 10^{-2} = 0 \tag{14}$$

Thus the corresponding polynomial can be written as:

$$P(z) = z^3 - 0.26z^2 + 5.98 \times 10^{-2} - 1.2167 \times 10^{-2} \quad . \tag{15}$$

The stability analysis of the covariance system that corresponds to the Discrete-Time Linear Time-invariant system will be addressed with respect to the characteristics equation(14) of the covariance system and the resulting polynomial in equation (15).The roots of the characteristics equation which are also referred to as the poles of the covariance system, strongly determines the stability of the covariance system. The Jury's stability criterion requires that the system poles are located inside the unit circle centered at the origin. Necessary conditions for stability will be examined before the sufficient condition will be examined in the Jury's table.

Necessary Condition

$$\left.\begin{array}{c} P(1) > 1 \\ (-1)^n P(-1) > 0 \\ |a_0| < |a_n| \end{array}\right\} \quad (16)$$

Sufficient Condition

$$\left.\begin{array}{c} |b_0| > |b_{n-1}|, \\ |c_0| > |c_{n-2}|, \\ \vdots \\ |q_0| > |q_2|. \end{array}\right\} \quad (17)$$

We will use the Jury's table format in Table 1(Fahri and Metin, 2019). The first row consists of the coefficients of $z^o, z^1, z^2, \dots, z^k, \dots, z^{n-2}, z^{n-1}, z^n$ in forward order while the second row is obtained by writing the first row in a reversed order. The third row is obtained by determinants where a_n and a_0 remains fixed for $b_{n-1}, b_{,n-2}, b_{n-3}, b_{n-4}, b_{n-5}, b_{n-1-k}, \dots, b_1, b_0$. The fourth row is obtained from the third row in reverse order.

Rows	z^0	z^1	z^2	z^3	… …		z^{n-2}	z^{n-1}	z^n
1	a_0	a_1	a_2	a_3	⋯⋯⋯⋯		a_{n-2}	a_{n-1}	a_n
2	a_n	a_{n-1}	a_{n-2}	a_{n-3}	⋯⋯	⋯	a_2	a_1	a_0
3	b_0	b_1	b_2	b_3	⋯⋯	⋯	b_{n-2}	b_{n-1}	
4	b_{n-1}	b_{n-2}	b_{n-3}	b_{n-4}	⋯⋯	⋯⋯	b_1	b_0	
5	c_0	c_1	c_2	c_3	⋯⋯	⋯⋯	c_{n-2}		

6	c_{n-2}	c_{n-3}	c_{n-4}	c_{n-5}	$\cdots\cdots$	$\cdots\cdots$	c_0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$			
2n-4	p_3	p_2	p_1	p_0			
2n-3	q_0	q_1	q_2				

Table 1: The Jury's Table

$$b_k = \begin{vmatrix} a_0 & a_{k+1} \\ a_n & a_{n-1-k} \end{vmatrix}, k = 0,1,2,\cdots\cdots, n-1$$

$$c_k = \begin{vmatrix} b_0 & b_{k+1} \\ b_{n-1} & b_{n-2-k} \end{vmatrix}, k = 0,1,2,\cdots\cdots, n-2 \qquad (18)$$

$$d_k = \begin{vmatrix} c_0 & c_{k+1} \\ c_{n-2} & c_{n-3-k} \end{vmatrix}, k = 0,1,2,\ldots\ldots, n-3$$

$$\vdots$$

Kalman's Controllability/Observability Conditions and Theorems

The system (5a) and (5b) is exactly controllable in some time T if and only if (Enrique, 2009):

$$rank[B \quad AB, \cdots, A^{n-1}B] = n. \qquad (19)$$

i.e if and only if the controllability matrix:

$$M_c = [B \;\; AB \;\; \cdots A^{n-1}B], \qquad (20)$$

is full row rank and (A, B) is a controllable pair.

For $A \in \mathcal{R}^{n\times n}$ and $C \in \mathcal{R}^{p\times n}$ in the system (5a) and (5b), the pair $((A, C)$ is ***observable*** if the observability matrix (Arnold, 2020; Erfan, 2020) :

$$O_b = \begin{pmatrix} C \\ CA \\ \vdots \\ CA^{\square-1} \end{pmatrix}, \qquad (21)$$

has LI columns, i.e., if rank $O_b = n$.

Thus, the covariance system (3.12) is controllable if and only if the controllability matrix;

$$M_c = [A_{c0v} \quad A_{cov}B_{cov} \quad {A_{cov}}^2 B_{cov}], \qquad (22)$$

has full row rank. Similarly, the covariance system is said to be observable if and only if the observability matrix:

$$M_O = \begin{pmatrix} C_{cov} \\ C_{cov}A_{cov} \\ \vdots \\ C_{cov}A_{cov}{}^{n-1} \end{pmatrix}, \qquad (23)$$

has a full row rank. Then, the pair $(A_{cov}C_{cov})$ is called an observable pair.

3. RESULTS AND DISCUSSION

The constant coefficients of the denominator polynomial (14) are examined for investigating necessary and sufficient conditions in (16) and (17)

The first step is to check for the necessary conditions.

$P(1) = 0.692367 > 0$
$a_n = -1.2167 \times 10^{-2}$ and $a_0 = 1$, implying $|a_n| < |a_0|$.
$(-1)^n P(-1) = 1.331967 > 0$. Thus, all necessary conditions are satisfied. We go further to investigate the sufficient condition for stability by constructing a jury's table.

In order to obtain the first row of the Jury's table write the constant coefficients of (15) in a forward order. Row two is obtained as a reversed order of row one. The elements of the third row are obtained as follows:

$b_0 = -9.999 \times 10^{-1}$, $b_1 = 2.5927 \times 10^{-1}$, $b_3 = -5.664 \times 10^{-2}$
The number of rows is 2n-3(rows), while the number of constraints is (n+1) constraints.

Thus, since the system is a third order system we have, 3 rows and 4 constraints.

Now we can form the Jury's table as shown below.

Rows	z^0	z^1	z^2	z^3
1	-1.2167×10^{-2}	5.98×10^{2}	-2.6×10^{-1}	1
2	1	-2.6×10^{-1}	5.98×10^{-2}	-1.2167×10^{-2}
3	-9.999×10^{-1}	2.5927×10^{-1}	-5.664×10^{-2}	

Table 2: Jury's table for the stability analysis of covariance system

Next we have to investigate the sufficient conditions for stability in the Jury's table

Check if $|b_0| > |b_{n-1}|$ i.e $|b_0| > |b_2|$
$|b_0| = -9.999 \times 10^{-1}$ and $|b_2| = -5.664 \times 10^{-2}$ which satisfies the condition that $|b_0| > |b_2|$.

Therefore, the covariance system is said to be stable because the necessary and sufficient conditions (i.e the 4 constraints) are completely satisfied.

Also, using the controllability matrix (22) we have,

$$A_{cov}B_{cov} = \begin{bmatrix} 1.2769 & 12.091 & 28.6225 \\ -0.2034 & -0.0929 & 4.1195 \\ 0.0324 & -0.2772 & 0.5929 \end{bmatrix}$$

$${A_{cov}}^2 = \begin{bmatrix} 0.0144 & 0.2184 & 0.8281 \\ -0.0084 & -0.0745 & -0.0814 \\ 0.0049 & 0.0158 & 0.0081 \end{bmatrix}$$

$${A_{cov}}^2 B_{cov=} \begin{bmatrix} 0.109561 & 2.433513 & 13.512976 \\ -0.049314 & -0.6734 & -1.39925 \\ 0.01993 & 0.02247 & 0.131553 \end{bmatrix}$$

So that the controllability matrix becomes:

$$M_C = \begin{bmatrix} 4 & 1.2 & 0.09 & 1.2769 & 12.091 & 28.6225 & 0.109561 & 2.433513 & 13.512976 \\ 0.2 & 8.03 & 1.2 & -0.2034 & -0.0929 & 4.1195 & -0.049314 & -0.6734 & -1.39925 \\ 0.01 & 0.8 & 16 & 0.0324 & -0.2772 & 0.5929 & 0.01993 & 0.02247 & 0.131553 \end{bmatrix}$$

The controllability matrix is full rank i.e $rank\ M_c = n$ where $n = 3$. Therefore, the system is controllable.

Similarly, using the observability matrix (21) we have,

$$C_{cov}A_{cov} = \begin{bmatrix} 0.25 & 1.3 & 1.69 \\ 0.01 & -0.04 & 0.04 \end{bmatrix} \quad , \quad C_{cov}{A_{cov}}^2 = \begin{bmatrix} 0.0144 & 0.2184 & 0.8281 \\ 0.0049 & 0.0126 & 0.0081 \end{bmatrix}$$

Thus, the observability matrix for the covariance system can be written as:

$$M_O = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0.25 & 1.3 & 1.69 \\ 0.01 & -0.04 & 0.04 \\ 0.0144 & 0.2184 & 0.8281 \\ 0.0049 & 0.0126 & 0.0081 \end{bmatrix}$$

rank $M_0 = 3$.Therefore,the covariance system is observable

Conclusion

The main objective of this research is to analyze the stability of the corresponding covariance system of a Discrete-Time Linear Time-Invariant Stochastic system in a Covariance Assignment Problem (CAP).The Jury’s criterion is an efficient method for analyzing stability of Discrete-Time Linear Time-Invariant System using the characteristics equation in the transfer function .In order to apply the Jury’s criterion, the constant coefficient of the characteristics equation which represents the transform of input signal of the covariance system ,were utilized. The Jury’s necessary and sufficient conditions for stability must be satisfied for a system to be called a stable system. The necessary conditions for stability were investigated in the characteristics equation while the sufficient conditions for stability of the covariance system were examined in the Jury's table. The covariance system proved to be a stable system which implies that all poles of the system are located within the unit circle in the z-plane.

Kalmans rank test proved to be a very efficient test for analyzing the controllability and observability of the covariance system in the CAP. For an n order system, the controllability/observability matrix must have full rank before we can conclude that the system is controllable/observable. The covariance system is controllable and also observable.

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