# ANALYSIS OF THE TICKETS TERMINAL OF KADUNA RAILWAY STATION: A QUEUING THEORY APPROACH

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**Abstract**
Queues are formed when different people require similar services, especially in the same place and at the same time interval. In this work, an analysis of the ticket terminal of Kaduna train station is carried out using Queuing theory. A single line multiple servers model for the system is developed and primary data were obtained from observation for the analysis. Results from the analysis shows that increasing the number of servers in the ticket terminal causes a decrease in the average waiting time of passengers in queue as well as in the system, thereby increasing passenger's satisfaction.

**Keywords -** Queuing Theory, Train station, Modeling in rail station, Waiting line, Railway station, controlling measures.

## 1.0 INTRODUCTION

Different people in various places may require similar services and sometimes in the same place, resulting in delay in being served in turn. These delays and waiting which may be in queue are problems common in almost every facets of life including banks, ticketing office, public transport systems, postal office, telecommunication, emergency services, computing, industrial engineering, project management among others. Queuing models are used as powerful tools for designing and evaluating the performance of queuing systems. Singh (2007) noted that any system in which arrivals of customers, place demand upon a finite capacity resource may be referred to as a queuing system. There are many studies in the literature that have shown the negative effects of queues on customers (Katz et al, 1991, Taylor, 1994, Hui and Tse, 1996).

Most service points attend to one customer at a time, before attending to the next in line. In doing so, others tend to wait for those who may have arrived at the service center before them to be served first, probably on a queue. With such arrangements, the number of queue may exceed one depending on the number of servers. Any customer coming to join an already existing queue may choose a queue of a server base on some factors which may include the length of the queue or length of services to be rendered as rightly noted by (Adan et al., 2000). According to Filipowiez (1999) and Kleinrock (1975), Queuing models are useful in providing basic framework for efficient design and analysis of several practical situations. Idzikowska (2000) and Lavenberg and Shedler (1975) in reaffirmation said Queuing models are essential for designing and monitoring of several communication systems.

Queuing theory has been used over the years to analyze queue length situations in different areas of life including railway stations. Yang et al. (2014) studied the passengers flow simulation in urban subway station based on Analogic process and software, considering Hangzhou station in China and found that optimizing the queuing system through of dynamical ticket windows in the station and at an affordable operation cost will help reduce the passenger queuing time. Long queues in most business areas reduces profit making as not all customers have the patients to wait in queue for a long time. Xu et al. (2007) analyzed the service performance of a ticket queue with balking customers, which focused on a ticket technology system and discovered that the probability of a balking varies on queues when customers' patience is low with high system traffic. They proposed improvement in the ticket system through online ticket booking. Talking about improvement through innovative and modern technology, Ghosal et al (2015) considered Android application for ticket booking and ticket checking in suburban railway. They proposed the use of modern tools like mobile phones, online ticketing and other modern devices to help reduce the stress of long queues in ticketing centers. Ituen-Umanah (2017) applied queuing theory to the ticket windows in railway stations, taking Lagos terminus as a case study. He analyzed the model using primary data from observation and discover that increasing the number of servers lead to a decrease in the average waiting time of passengers in queue and a decrease in the average waiting time of passengers in the system.

Railway transportation is one among the major forms of land transportation that uses wheeled vehicles running on railroad in conveying large number of people and goods from one place to another. Though not commonly used, most people prefer the railway transportation to other forms of land transportation because of its high level of safety and affordability. In 1955, the Nigerian government established and passed it into law the Nigerian Railway Corporation (NRC), an act that gives license to build and operate rail stations in the country (Akwara, Udaw & Ezirim, 2014). Even though rail transportation has been in existence in Nigeria prior to 1955 (Ituen-Umanah, 2017) it became more recognized and acceptable after 1955. NRC now owns and operate rail stations across Nigeria where passengers board trains to their destinations. Kaduna railway station is one among the railway stations established by the NRC spread across Nigeria, with terminal in Kaduna state of Nigeria. This railway station is designed for activities that aid in the process of rail transportation. One among the most important aspect of this activity is the ticket section where all passengers are required to purchase such tickets before securing their seats. Modern technology has made it possible for some passengers to purchase tickets online, however, there is always the need to present evidence of such booking before accessing the train. This series of activities around the ticket terminal creates room for long queue at the train station.

This work will review queuing theory and study the Kaduna train station. It will be carried out to detect the variability in a quality of service at the train station, and also find the average queue length passengers see before gaining access to the train in order to improve on the services at the train station, especially as it involves increasing passengers' satisfaction. Lastly, it will obtain results that will be time-dependent. These results will determine whether or not the services at the Kaduna train station is satisfactory or will need improvement.

The objective of this study is to analyze the tickets terminal of Kaduna railway station using Queuing theory to better improve the passengers' satisfaction. Specifically, the study sought to:

i. observe the queue system at the ticket terminal of the Kaduna railway tickets terminal.
ii. develop an appropriate model for the observed problem.

iii. determine statistical results from analysis as well as the performance measures for the model for an improved service delivery.

iv. compare results from analysis with an increased assumption.

v. make conclusions on the derived comparison.

## 2.0 MATERIALS AND METHOD

The type of research used in this particular study is a case study design that examines a particular case. This study uses primary data collected from the case study through observation of the queuing system. The instruments used in collecting these data includes a pen, a stop watch, a movable chair and table, a recording sheet. All passengers arrived at the arrival point to purchase tickets and/or gain access into the train together with the servers who issue out the tickets or clearance make up part of the population, Data was collected for a period of 14 days from Monday to Sunday of two weeks between working hours when there is an awaited train or a train within the station is about to be loaded.

Kaduna train station runs a four trips per day schedule through its routes, as at from 2018 with Kaduna to Abuja as its major route, even though Sundays and Wednesdays are exceptions with only two trips. Other days of the week are normal schedules. For Sundays, Departure from Idu station is 12:30pm for first trip and 5:00pm second trip. Arrival at Kubwa station is 12:44pm for first trip and 5:10pm for second trip. Departure from Kubwa station is 12:49pm for first trip and 5:15pm for second trip. Arrival at Risaga station is 3:00pm for first trip and 7:05pm for second trip. For Wednesday schedule, we have departure from Rigasa station at 6:40pm for first trip and 6:00pm for second trip. Arrival at Kubwa station is 8:28am for first trip and 8:05pm for second trip. Arrival at Idu station station is 8:43am for first trip and 8:20pm for second trip.

Kaduna to Abuja schedules for other days we have departure from Rigasa is 6:40am, 10:35am, 2:00pm and 6:00pm for first, second, third and fourth trips respectively. Arrival at Kubwa is 8:28am, 12:59pm, 3:58pm, and 8:04pm for the respective first, second, third and fourth trips. Departure from Kubwa is 8:21am, 2:04pm, 4:03pm and 8:07pm for first, second, third and fourth trips respectively then arrival at Idu is 8:43am, 1:20pm, 4:16pm and 8:20pm for first, second, third and fourth trips respectively.

## 3.0 MODEL FORMULATION AND SPECIFICATION

The particular models to represent the system we are working on is being specified. This model is a description of the way and manner our queuing system behaves, based on the situation we are dealing with. The appropriate queuing model for this study was established using the method as used in Kendall (1953);

The Model is given by

$$A/S/c/\infty/\infty/FIFO$$

Where $K = \infty, n = \infty$ and the queue discipline follow the first in first out discipline.

For the purpose of the Marcovian or Poisson process, we can write our model as

$$M/M/c/\infty/\infty/FIFO$$

For this particular model, certain assumptions were generally made to guide the whole process. Such assumptions include;

i. Arrival process follow a Poisson process
ii. No arrival left the queue without being served.
iii. Every passenger arrives at the queue independently at the same arrival rate
iv. The servers used were all identical
v. Service times were exponentially distributed
vi. The system capacity has no limit
vii. Passengers were served on a First-in-First-Out basis

## 4.0 THE M/M/c MODEL

The M/M/c queuing model assumes a single queue with unlimited waiting room that feeds into c identical servers. For the model formulated above, we assume the following working parameters;

(i) Arrival Rate: Rate at which the passengers are served at the train station. It is denoted by $\lambda$ and given as $\lambda = \frac{Number\ of\ passenger\ that\ arrived(Npa)}{time\ of\ observation(To)}$

(ii) Number of Servers: The queuing calculations depends on whether there is a single server or multiple servers for the queue. A single server queue has one server for the queue while a multiple server queue corresponds to a situation such as the number of attendants giving out tickets at a ticket stand. The number of server is denoted by c

(iii) Service Rate: Number of passengers served per unit time. It is denoted by $\mu$ and given as $\mu = \frac{Number\ of\ passenger\ served\ (Nps)}{time\ of\ observation (To)}$

(iv) Traffic Intensity: The queue intensity is the ratio of the total number of customers who join the system and the total number of customers who are served by the three channels in a given time period. If the ratio is more than 1 it means the queue intensity is high and if the ratio is less than 1 it means the queue intensity is low. If the queue intensity is equal to 1 it means the queue is normal and evenly distributed. It is denoted by $\rho$ and given by the expression below as $\rho =$ traffic intensity = $\frac{\lambda}{c\mu}$,

Working with number of passengers in a railway station we can modify the above formula to incorporate more parameters in getting what we want.

Therefore,

i. The probability that the system is idle is denoted by $P_0$ and is given by

$$\rho_o = \left[\sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!} + \frac{1}{c!} \cdot \frac{(c\rho)^c}{(1-\rho)}\right]^{-1} \quad (1)$$

where

$r$ = Number of servers

$\lambda$ = Average arrival rate

$\mu$ = Average service rate

$\rho$ = Traffic intensity or Utilization rate

ii. The average number of passengers in queue is denoted by $L_q$ and is given by the expression below

$$L_q = \left[\frac{1}{(c-1)!}\left(\frac{\lambda}{\mu}\right)^c \frac{\lambda\mu}{(c\mu-\lambda)^2}\right]\rho_o \quad (2)$$

iii. The average number of passengers in the system is denoted by $L_s$ and is given by

$$L_s = L_q + \frac{\lambda}{\mu} \quad (3)$$

iv. The average waiting time for a passenger in queue is denoted by $W_q$ and is given by

v. $$W_q = \frac{L_q}{\lambda} \quad (4)$$

vi. The average waiting time for a passenger in the system is denoted by $W_s$ and is given by

$$W_s = W_q + \frac{1}{\mu} \quad (5)$$

Equations 1 – 5 are used to calculate parameters for the model of this study. The data collected and calculated are presented in Table 1-4. Table 1 shows the arrival time and departure time of customers at Kaduna train station.

**Table 1: Summary and Presentation of primary data**

| Activity | Arrival of Passengers | Number served | Time of observation(Hrs) | Number of Servers (c) |
|---|---|---|---|---|
| Day 1 | 413 | 321 | 4 | 2 |
| Day 2 | 337 | 311 | 3 | 2 |
| Day 3 | 387 | 307 | 3 | 2 |
| Day 4 | 391 | 379 | 3 | 2 |
| Day 5 | 381 | 299 | 2 | 2 |
| Day 6 | 291 | 189 | 3 | 2 |
| Day 7 | 191 | 111 | 3 | 2 |
| Day 8 | 424 | 331 | 3 | 2 |
| Day 9 | 392 | 367 | 3 | 2 |
| Day 10 | 377 | 308 | 2 | 2 |
| Day 11 | 301 | 279 | 3 | 2 |
| Day 12 | 371 | 293 | 3 | 2 |
| Day 13 | 301 | 273 | 3 | 2 |
| Day 14 | 211 | 192 | 2 | 2 |

The calculated results are presented in the table 2 below;

**Table 2: Presentation of results from primary data**

| Days | $\lambda$ | μ | Time of observation | $\rho$ | $P_0$ | $L_q$ | $L_s$ | $W_s$ | $W_q$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 103 | 80 | 4 | 0.6438 | 0.2167 | 0.0907 | 1.3782 | 0.0008 | 0.0133 |
| 2 | 112 | 104 | 3 | 0.5385 | 0.3000 | 0.4397 | 1.5166 | 0.0039 | 0.0135 |
| 3 | 129 | 102 | 3 | 0.6324 | 0.2252 | 0.8426 | 2.1073 | 0.0065 | 0.0163 |
| 4 | 130 | 126 | 3 | 0.5159 | 0.3193 | 0.3740 | 1.4057 | 0.0029 | 0.0108 |
| 5 | 191 | 150 | 2 | 0.6367 | 0.2220 | 1.0304 | 2.3371 | 0.0052 | 0.0723 |
| 6 | 97 | 63 | 3 | 0.7698 | 0.1301 | 2.2411 | 3.7808 | 0.0023 | 0.0182 |
| 7 | 64 | 37 | 3 | 0.8649 | 0.0724 | 51.2941 | 53.0238 | 0.8015 | 0.8285 |
| 8 | 141 | 110 | 3 | 0.6409 | 0.2233 | 0.9118 | 2.1936 | 0.0065 | 0.0156 |
| 9 | 131 | 122 | 3 | 0.5369 | 0.3013 | 0.4348 | 1.5086 | 0.0033 | 0.0115 |
| 10 | 189 | 154 | 2 | 0.6136 | 0.2395 | 0.7415 | 2.0145 | 0.0039 | 0.748 |
| 11 | 100 | 93 | 3 | 0.5376 | 0.3007 | 0.5055 | 1.5808 | 0.0051 | 0.0159 |
| 12 | 124 | 98 | 3 | 0.6327 | 0.2250 | 0.8444 | 2.1097 | 0.0068 | 0.017 |
| 13 | 100 | 91 | 3 | 0.5495 | 0.2907 | 0.4751 | 1.5793 | 0.0048 | 0.0158 |
| 14 | 106 | 96 | 2 | 0.5508 | 0.3968 | 0.6657 | 1.7699 | 0.0063 | 0.0167 |

We used the same method in the analysis of the whole 14 days to give a reasonable consideration to the queuing system, through averages of the total values presented. The results of the analysis with the averages will provide a concise decision for the total number of days used. These results are given below;

**Table 3: Summary showing results of analysis of 14 days**

| Inputs | Results for 14 days |
|---|---|
| Number of passengers arrived in 14 days | 4768 |
| Number of passengers served in 14 days | 3960 |
| Number of time of observation in 14 days | 40 |
| $c$ | 2 |
| $\lambda$ | 119 |
| $\mu$ | 99 |
| $\rho$ | 0.6010 |
| $Po$ | 0.2072 |
| $Lq$ | 0.5651 |
| $Ls$ | 1.7671 |
| $Wq\ (hours)$ | 0.0047 |
| $Ws\ (hours)$ | 0.0148 |

From our findings, the arrival rate $\lambda$ is greater than the service rate $\mu$, which indicates that a queue exist. The type of queuing model used in this study is the M/M/c queuing model, where c represents the number of servers. In this study, we have only 2 servers which gives us the average waiting time of passengers in queue, $W_q$ as 0.0047 (hours) while the average waiting time of passenger in the system $W_s$ as 0.0148 (hours). Assuming we increased the number of servers to say 3, we will have different average waiting time for passengers on queue and in the system.

We can comprehensively present these results on a tabular form in the table 4 below;

**Table 4: Results from increase in number of servers**

| Inputs | c= 2 | c= 3 | c= 4 |
|---|---|---|---|
| $\lambda$ | 119 | 119 | 119 |
| $\mu$ | 99 | 99 | 99 |
| $\rho$ | 0.6010 | 0.4007 | 0.3005 |
| $Po$ | 0.2072 | 0.2934 | 0.2996 |
| $Lq$ | 0.5651 | 0.0947 | 0.0160 |
| $Ls$ | 1.7671 | 1.2967 | 1.218 |
| $Wq\ (hours)$ | 0.0047 | 0.0008 | 0.0001 |
| $Ws\ (hours)$ | 0.0148 | 0.0109 | 0.0102 |

## 5.0 Discussion

Observations from the increase in the number of servers from two (2) to three (3) show a decrease in the average waiting time of a passenger in queue from 0.0047 hour to 0.0008 hour. We also observed that the average waiting time of passenger in the system decreased from 0.0148 hour to 0.0109 hour. Further reduction of the waiting time of passenger in queue and in the system was also observed when the number of servers increased from three (3) to four (4) servers. The waiting time of passengers in queue reasonably decreased from 0.0008 hours to 0.0001hour while the waiting time of passengers in the system also decreased from 0.0109 hours to 0.0102 hours. These results are very important to this study as they bring meaning and satisfaction to passengers, since their queue can be improved to reduce stress of waiting in long queue.

## 6.0 Conclusion

This study analyzed the tickets terminal of Kaduna train station using queuing theory, where more emphasis was made on the relevance of queuing theory in reducing waiting line problems. Results of the analysis showed that the waiting time of passengers can be improved upon to ensure maximum satisfaction. It also showed that their servers are not restricted but flexible for better results. It can be concluded from the findings in the results that the management of the Kaduna railway station and other railway stations management can use proper analysis of queuing theory to make improved and better decisions to increase customer satisfaction and also make more profits through reduced queue length which guarantee more services within the shortest length of time. More number of servers can be created at different points to reduce queue length and also, modern devices and platforms can help in that regards. It is expected that, the Nigerian Railway Corporation should require the services of trained Operations Research consultants to carry out periodic analysis of the queuing situations at the railway stations. More number of servers should be added to gain maximum satisfaction and buying of tickets at railway stations can be made through modern platforms, thereby reducing the work at the railway terminals to confirming already purchased tickets online, which will reduce queue length.

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