ANALYSIS OF TICKETING SERVICES OF NIGERIA RAILWAY CORPORATION USING M/M/S QUEUING MODEL: A CASE STUDY OF UJEVWU AND ITAKPE TRAIN STATIONS

Judith E, Nicholas[1] and John N. Igabari[2]
Department of Mathematics, Delta State University, Abraka, Nigeria
Corresponding Authors: nicholas.judith@delsu.edu.ng; jn_igabari@delsu.edu.ng

Abstract

The study was carried out to evaluate the queuing activities at ticket windows of two selected train terminals with a view to determining their performances and improving customer service while minimizing the economic cost of operating both stations. We observed the arrival and departure patterns of passengers at the ticket section for a period of 24 days for each station within the hours of 6 am-8 am and 1 pm-3 pm for Ujevwu and Itakpe terminals respectively. The M/M/S model with poisson and exponential processes was used to derive the arrival rate and service rates, utilization factor, waiting times of passengers in the system (queue and in-service), and the probability of n customers in the system. It was observed that passengers arrive at the Ujevwu ticket section at an average rate of 1.184 persons per minute and are serviced at the rate of 0.894 per minute. Also, the rates of arrival of passengers at the Itakpe station were calculated to be 2.092 per minute and service was at 1.787 persons per minute. A sensitivity analysis was done for both systems for when the service channels are increased to three and four servers to ascertain which best optimizes the overall system. The system were both found to be at their best with an increase in service channels from two to three and switching between three and four servers at peak days, particularly for the Itakpe station.

Keywords: Ticketing Services, Railway Corporation, Queues, M/M/S Model, Poisson, and Exponential Processes.

1. Introduction

Railway transportation is a major form of land-based transportation that conveys large number of people and goods, using wheeled vehicles (trains and trams) that run on a track known as a railroad or railway (Ituen-Umanah, 2017). Railway transportation offers huge potentials because of its advantages like relative safety, dependability, affordability and the capability of transforming an economy through mass movement of people, goods and services (Igwe *et al*., 2013, Gregus et al, 2022). In Nigeria, railway transportation has been in existence since 1898, however in 1955, an Act establishing the Nigerian Railway Corporation (NRC) – giving it the right to construct and operate railway services in Nigeria – was passed into law (Akwara, Udaw & Ezirim, 2014). Nigeria railway corporation also own and operate railway stations across the country. A railway station basically consists of at least one-track side platform and a station building, inside the station building, various operations are carried out. The Ticketing operation is one of the activities that are carried out in the railway station and also one of the most important operation as well, as tickets purchased by passengers provides them access to services desired; while payments received from passengers form huge portions of revenue accrued from services offered by the station in stations like the Ujevwu and Itakpe Terminus, ticketing officers attend to passengers through barricaded windows known as "ticket windows". In some railway stations, ticket windows are not sufficient and where enough windows are physically available, all may not be functional, resulting in long queues. There are also observed cases where passengers are not seen at ticket windows for long periods, creating a situation where servers are idle for such periods. A lack of orderliness, owing to ticketing officers attending first to passengers they know, at the expense of other passengers that arrived earlier and cases of negative passenger behaviour (like collusion or jockeying), increase the waiting line problem faced at ticket windows. Waiting for a particular service is a common experience in the transportation industry; the time spent for waiting is related to service assessment (McCord et al, 2010). Train delays are often considered by passengers as an indication of poor services (Berger *et al*, 2011) a long delay can negatively affect overall service evaluation and increase the severity of negative effective responses such as anger because of the uncertainty created by delays (Patel et al , 2012). Therefore, delays are major reasons for public transportation complaints. As such, many railway companies have increased their efforts to eliminate delays. Queuing theory is known as Random System Theory which has the solutions for statistical interference and problem of behaviour and optimization in queuing system (Kavitha & Palaniammal, 2014). It is the mathematical study of waiting lines (Sundarapandian, 2009) which has now become an area of scientific inquiry, sub-discipline within operations research (Cope *et al*., 2011). Queues are a common sight in our everyday life and they are formed whenever the demand for services is higher or surpasses the capacity to serve them. In other words, a queue can be generated if the number people or things requiring a particular service outweigh available capability of servers to function. Queues can be encountered in various ways from purchasing tickets at the train station, at banking halls: waiting to make a bank deposit to patients waiting at clinics. Many organizations such as banks, airlines, health care systems, telecommunications companies and security departments routinely use queuing theory models to help determine capacity levels needed to experienced demands in a more efficient way (Peter & Sivasamy, 2019).However, most customers are not comfortable with waiting or queuing (Olaniyi, 2004). Undoubtedly, there are numerous factors that affect a customer's perception of the waiting experience, some of which include: physical, psychological and emotional. If there were to be no queue at all, it would create the impression that the value of the attraction is to some extent diminished. The delays and chaotic scenario associated with waiting in a queue in order to have access to desired service are not only experienced by people, as even things, signals also can be faced with situations of queue. While it has been seen that patronage of railway services have reduced in recent times due to high operational cost, lack of the right management attitude and inadequate funding of this sector in Nigeria, a fact corroborated by Agunloye and Odunwaye, (2011), in a study of factors that has influenced the quality of railway transportation services in Lagos metropolis, among other things, the study revealed that 80% of railway patrons agreed that the services rendered were ineffective and inadequate. From the study it is easy to ascertain that poor services at the ticket window of the railway station constitute a part of the problems that has influence the quality of the Nigeria rail transport sector, especially when it comes to the queuing situation. A study carried out by Tari *et al*. (2018) in Nigeria at the Idu railway station on the queuing situation at its ticket purchasing windows revealed that there is a positive correlation between an increase in service facility and customer's waiting time. This study is intended to apply the principles of queuing models to the Ujevwu/Itakpe terminus in Delta state and Kogi state with a view to enhance efficiency and customer satisfaction. A key fact that is worth establishing is that the railway terminus is not fully organized hence it requires programming methods and scheduling models for its enhancement.

2. The Queuing Model

The multi-server queuing model with poisson arrivals and exponential service time and a queue discipline of first-come-first-served (M/M/S/∞/ FCFS) was used in this study (Taha, 2007; Hillier and Lieberman, 2015). For the model the following assumptions are made;

- The arrival of passengers follows a poisson process
- Passengers arrivals were independent and the arrival rate is constant
- A single line was formed and passengers arriving were assumed to be patient customers who wait for their service to be delivered regardless of the length of the queue
- Service times were exponentially distributed and the mean service rate was constant for each server
- The station had identical servers in parallel
- A passenger does not leave the queue until service is rendered
- The queue discipline is first-come-first-serve
- Infinite capacity of the system

2.1 Multi-Server Queuing Model (M/M/S FCFS)

In this model, there are multiple but identical servers as opposed to a single server and these servers are in parallel to handle arriving passengers. This queuing system assumes the poisson arrival at an average rate of **λ** passengers, per unit time and is serviced on a first come first served basis by any of the servers. The service times are distributed exponentially with an average of **μ** passengers per unit of time.

The following two cases may arise if there are n customers in the queuing system at any point in time;

- If n<s (number of passengers in the system is less than numbers of servers) then there will be no queues. However, (s-n) numbers of servers are not busy. The combined service rate will be $\mu_{n=n}\mu$; $n < s$
- If $n \geq s$(number of passengers in the system is more than or equal to the number of servers) then all servers will be busy and the maximum number of passengers in the queue will be (n-s). The combined service rate will b0e $\mu_n \ = s_\mu$; $n \geq s$

Thus to derive the results for this model, we have;

$\lambda_n = \lambda$ for all $n \geq 0$

$$\mu_n = \begin{cases} n\mu & n < s \\ s\mu & n \geq s \end{cases}$$

According to (Sharma, 2013), the method of determining the probability $P_n$of n passengers/customers in the queuing system at time t and the value of its various characteristics is summarized below;

Let $P_n(t)$ be the probability that there are n passengers in the system at time t. The differential difference equation of the model are;

$$\frac{d}{dt}P_0(t) \ = -\lambda P_0\,(t) \ + \mu P_1(t)\,;\, n = 0$$

$$\frac{d}{dt}P_n(t) = \ -(\lambda + n\mu)P_n(t) + \ \lambda P_{n-1}\,(t) + \ (n+1)\mu P_{n+1}(t)\,; \quad 1 \leq n < s$$

$$\frac{d}{dt}P_n(t) \ = \ -(\lambda + s\mu)P_n(t) \ + \ \lambda P_{n-1}\,(t) + \ s\mu P_{n+1}\,;\, n \geq s$$

In steady –state, limit as $t \to \infty$ $P_n(t) \ = \ P_n$limit as $t \to \infty$ $\frac{d}{dt}P_n(t) \ = 0$

Therefore, we have

$$0 \ = \ -\lambda P_0 + \ \mu P_1; \qquad n = 0 \tag{1}$$

$$0 \ = \ -(\lambda + n\mu)P_n \ + \ \lambda P_{n-1} \ + \ (n+1)\mu P_{n+1}; \quad 1 \leq n \leq s \tag{2}$$

$$0 \ = \ -(\lambda + s\mu)P_n + \ \lambda P_{n-1} \ + \ s\mu P_{n+1}; \qquad n \geq s \tag{3}$$

From (1), we have

$$P_1 = \left(\frac{\lambda}{\mu}\right)P_0 \tag{4}$$

From (2) we have

$$(n+1)\mu P_{n+1} = \ \lambda P_n + \ n\mu P_n - \lambda P_{n-1} (5)$$

Putting $n = 1$in (5) ,we get

$$2\mu P_2 = \lambda P_1 + \mu P_1 - \lambda P_0 = 0$$

$$P_2 = \frac{\lambda}{2\mu} P_1 = \frac{1}{2!}\left(\frac{\lambda}{\mu}\right)^2 P_0 \tag{6}$$

Putting n = 2 in (5) we get

$$3\mu P_3 = \lambda P_2 + 2\mu P_2 - \lambda P_1 = 0$$

$$P_3 = \frac{\lambda}{3\mu} P_2 = \frac{1}{3!}\left(\frac{\lambda}{\mu}\right)\left(\frac{\lambda}{2\mu}\right)^2 P_0 \tag{7}$$

$$P_3 = \frac{1}{3!}\left(\frac{\lambda}{\mu}\right)^3 P_0$$

$$\vdots$$

$$P_n = \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0; \qquad 1 \le n < s \tag{8}$$

When $n \geq s$

If $n = s - 1$, we get from (5) ;

$$S\,\mu P_s = \lambda P_{s-1} + (s-1)\mu P_n - \lambda P_{s-2} = 0$$

$$P_s = \frac{\lambda}{sn} P_{s-1} \quad and \quad P_{s-1} = \frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^{s-1} P_0$$

$$P_s = \frac{1}{s}\left(\frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^{s-1} \frac{1}{(s-1)!} P_0$$

$$P_s = \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s P_0 \tag{9}$$

When $n = s$ in (5), we get

$$S\mu P_{S+1} = \lambda P_S + s\mu P_S - \lambda P_{S-1} = 0$$

$$P_{S-1} = \frac{\lambda}{S\mu} P_S$$

$$P_{S+1} = \frac{\lambda}{S\mu}\frac{1}{S!}\left(\frac{\lambda}{\mu}\right)^S P_0$$

$$P_{S+1} = \frac{1}{S}\frac{1}{S!}\left(\frac{\lambda}{\mu}\right)^{S+1} P_0 \tag{10}$$

When $n = s + 1$, from (5), we get

$$S\mu + P_{S+2} = \lambda P_{S+1} + S\mu P_{S+1} - \lambda P_S = 0$$

$$P_{S+2} = \left(\frac{\lambda}{S\mu}\right) P_{S+1}$$

$$= \frac{1}{S}\frac{\lambda}{\mu}\frac{1}{S}\frac{1}{S!}\left(\frac{\lambda}{\mu}\right)^{S+1} P_0$$

$$P_{s+2} = \frac{1}{S^2}\frac{1}{S!}\left(\frac{\lambda}{\mu}\right)^{S+1} P_0$$

$$\vdots$$

$$P_S + (n-s) = P_n = \frac{1}{s^{n-s}}\frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^n P_0 \; ; \; n \geq s \tag{11}$$

To obtain P_0 , we use the condition;

$$\sum_{\mathrm{n}=0}^{\infty} \mathrm{P_n} = \sum_{n=0}^{s-1} \mathrm{P_n} + \sum_{\mathrm{n=S}}^{\infty} \mathrm{P_n} = 1 \tag{12}$$

$$\sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0 + \sum_{n=s}^{\infty} \frac{1}{s^{n-s}\, s!}\left(\frac{\lambda}{s\mu}\right)^n P_0 = 1 \tag{13}$$

$$\mathrm{P_0}\left[\sum_{\mathrm{n}=0}^{\mathrm{s}-1} \frac{1}{\mathrm{n}!}\left(\frac{\lambda}{\mu}\right)^{\mathrm{n}} + \frac{\mathrm{s^s}}{\mathrm{s}!}\sum_{\mathrm{n=s}}^{\infty} \frac{1}{\mathrm{s^n}}\left(\frac{\lambda}{\mu}\right)^{\mathrm{n}}\right] = 1$$

$$\mathrm{P_0} = \left[\sum_{\mathrm{n}=0}^{\mathrm{S}-1} \frac{1}{\mathrm{n}!}\left(\frac{\lambda}{\mu}\right)^{n} + \frac{\mathrm{s^s}}{\mathrm{s}!}\sum_{\mathrm{n=s}}^{\infty} \frac{\mathrm{s^n}}{\mathrm{s^n}}\left(\frac{\lambda}{\mathrm{s}\mu}\right)^{\mathrm{n}}\right] = 1$$

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right) n + \frac{s^s}{s!}\sum_{n-s}^{\infty} \left(\frac{\lambda}{s\mu}\right)^{n}\right] = 1 \tag{14}$$

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n}\left(\frac{\lambda}{\mu}\right)^n + \frac{s^s}{s!}\left(\frac{\lambda}{s\mu}\right)^s + \left(\frac{\lambda}{s\mu}\right)^{s+1}\left(\frac{\lambda}{s\mu}\right)^{s+2} + \ldots\ldots\right] = 1 \quad (15)$$

$$P_0 = \left[\sum_{n=0}^{S-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{s^s}{s!}\left[\left(\frac{\lambda}{s\mu}\right)^s\left[1 + \left(\frac{\lambda}{s\mu}\right) + \left(\frac{\lambda}{s\mu}\right)^2 + \ldots\ldots\right]\right]\right] = 1$$

$$P_0 = \left[\sum_{n=0}^{S-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{S^S}{S!}\frac{\lambda^S}{S^S\mu^S}\left[\frac{1}{1-\left(\frac{\lambda}{S\mu}\right)}\right]\right] = 1$$

$$P_0 = \left[\sum_{n=0}^{S-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s\left(\frac{s\mu}{s\mu-\lambda}\right)\right] = 1$$

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s\left(\frac{s\mu}{s\mu-\lambda}\right)\right] - 1 \quad (16)$$

2.2 Performance Measures for the Model

Expected number of customers waiting in queue (length of line)

$$L_q = \sum_{n=s}^{\infty}(n-s)\,P_n = \sum_{n=s}^{\infty}(n-s)\frac{P^n}{s^{n-s}s\,!}P_0 \quad (17)$$

$$= \frac{\rho^S p_0}{S\,!}\sum_{n=s}^{\infty}(n-s)\,\rho^{n-s}\,;\;\; \rho = \frac{\lambda}{\mu} \quad (18)$$

$$= \frac{\rho^S P_0}{S}\sum_{m=0}^{\infty} m\rho^m \qquad ; n-s = m \quad (19)$$

$$= \frac{\rho^S}{S\,!}.\rho P_0\sum_{m=0}^{\infty} m\rho^{\mathrm{m}-1} = \frac{\rho^s}{s\,!}\,.\rho P_0\frac{d}{d\rho}\left[\sum_{m=1}^{\infty}\rho\, m\right]$$

$$= \frac{\rho^S}{S!}.\rho P_0\frac{1}{(1-\rho)^2} = \left[\frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s\frac{\lambda\,.s\mu}{(s\mu-\lambda)^2}\right]P_0$$

$$= \left[\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^{s} \frac{\lambda\mu}{(s\mu-\lambda)2}\right] P_0 \tag{20}$$

Expected number of customers in the system

$$L_s = L_q + \frac{\lambda}{\mu} \tag{21}$$

Expected waiting time of a customer in queue

$$W_q = \left[\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^{s} \frac{\mu}{(s\mu-\lambda)^2}\right] P_0 = \frac{L_q}{\lambda} \tag{22}$$

Expected waiting time that a customer spends in the system

$$W_s = W_q + \frac{1}{\mu} = \frac{L_q}{\lambda} + \frac{1}{\mu} \tag{23}$$

Probability that all servers are simultaneously busy (utilization factor)

$$P(n \geq s) = \sum_{n=s}^{\infty} P_n = \sum_{n=s}^{\infty} \frac{1}{s^{n-s} s\,!}\left(\frac{\lambda}{\mu}\right)^{n} P_0 \tag{24}$$

$$= \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^{s} P_0 \sum_{m=0}^{\infty}\left(\frac{\lambda}{\mu}\right) = \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^{s} \frac{s\mu}{s\mu-\lambda} P_0 \qquad (24)$$

Probability that a passenger enters the system without waiting

$$P(n < s) = 1 - P(n \geq s) \tag{25}$$

3. Description of Study Area

The Ujevwu-Itakpe Railway is the railway connecting the port city of Warri to Itakpe. Its construction began in 1987 as an industrial railway as a cargo train to supply the Ajaokuta Steel Mill iron-ore and coal. After a period of construction that lasted for more than 30 years, the railway was in 2020 officially inaugurated by the Minister of Transport as a passenger line.

The Ujevwu- Itakpe train terminus is a very busy terminal with a high influx of passengers on a daily basis, this high rate of passenger’s patronage has led to the inability of train management to be able to offer quality services, as most times the service channels have no enough servers to adequately handle all passengers in due time leading to delay in train departure time. The Itakpe station has 5 ticket windows with only 2 operational windows to service passengers who have come to board the train. The Ujevwu terminus also has 2 ticket windows which are fully operational and service passengers on the queue for tickets to access the train.

4. Data Collection Procedure

Input data was collected two (2) hours before the train's departure from 06:00hrs to 08:00hrs in the morning for a period of 24 days for the Ujevwu Station and two (2) hours before the departure time for the Itakpe train during the afternoon from 1:00HRS to 3:00HRS by observation during the peak periods and off-peak periods. This was done to enable the averages of the arrival rate and the service rates to be determined at every travelling time, Using a stopwatch and a recording paper. The population for this study is made up of all passengers that come to the Itakpe and Ujevwu terminus from an unlimited population source to purchase train tickets for travelling and equally all servers that facilitate the ticketing operations .the sample of the study was suitably drawn from all passengers entering the station and servers involved in the ticketing process. An M/M/S queuing software was used in analyzing the data obtained.

5. Data Presentation and Analysis

The observed passenger's arrival and departure patterns at the ticketing sections were presented for two selected train terminals together with the results of statistical analysis of survey data and calculations of data using queuing model parameters. A queuing software R Programming version 4.2.1 was used for analysis to save computation time and allow flexibility in determining the various performance measures of the system (Itakpe, Ujuvwe train terminals) such as average time spent in the system, (both on the queue and in service) in respect to the various departure time.

The data collected via direct observation indicated the following metrics:

Table 1: **Arrival & Departure patterns for passengers at Itakpe train station**

	DAILY ARRIVAL	DAILY DEPARTURE
	186	145
	205	182
	209	180
	222	185
	223	172
	239	190
	219	186
	266	211
	232	195
	255	219
	293	244
	308	272
	215	180
	229	181
	220	196
	287	256
	322	299
	314	280
	228	196
	256	222
	213	180
	223	178
	326	294
	327	303
DAILY RATE	250.7	214.4
HOURLY	125.4	107.2
MINUTE	2.089	1.787

Table 2: Arrival & Departure patterns for passengers at Ujevwu train station

	DAILY ARRIVAL	DAILY DEPARTURE
	179	91
	132	82
	121	76
	120	79
	164	129
	179	147
	135	94
	121	82
	108	89
	106	76
	181	139
	175	137
	123	93
	129	92
	105	86
	108	89
	181	168
	208	166
	101	72
	102	76
	123	96
	129	103
	176	140
	205	172
DAILY RATE	142.1	107.3
HOURLY	71.06	53.63
MINUTE	1.184	0.894

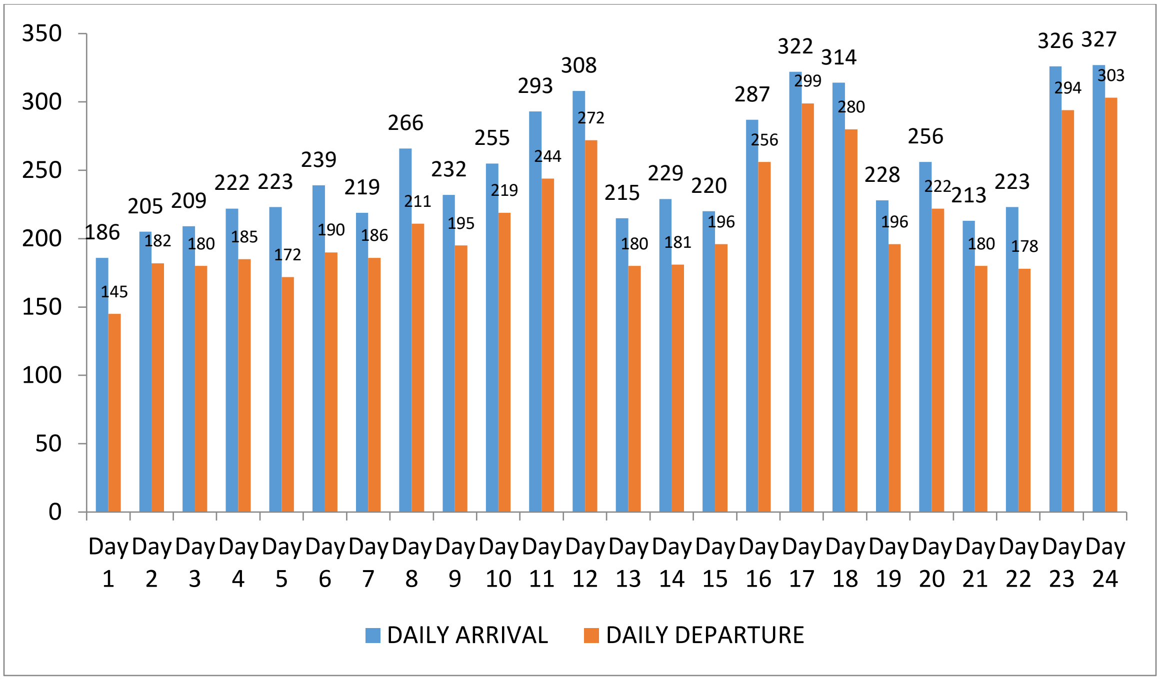

Figure 1: Arrival & Departure patterns for passengers at Itakpe train station

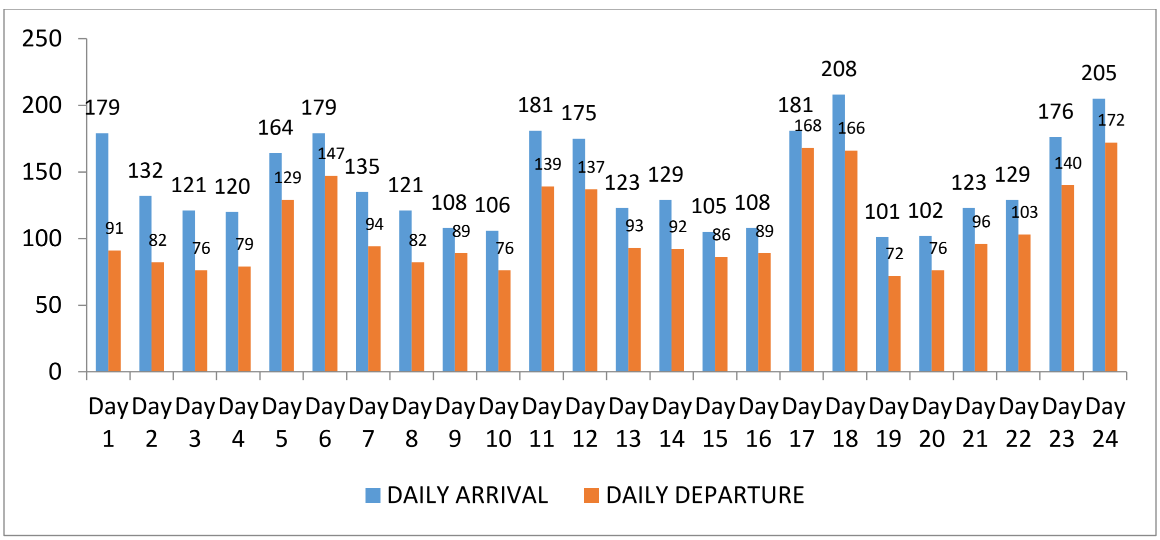

Figure 2: Arrival & Departure patterns for passengers at Ujevwu train station

Table 3: Summary of analysis result Ujevwu train station

M/M/2Queueing Calculations				
Basic Parameters			**State Probabilities**	
Arrival Rate	**1.184**	per min	Number in system	Probability
Service Rate	**0.894**	per min	**0**	**20%**
Number of Servers	**2**		**1**	**27%**
Time Unit	**Min**		**2**	**18%**
			3	**12%**
Basic Performance Measures			**4**	**8%**
Utilization	**66%**		**5**	**5%**
P(0), probability that the system is empty	**20%**		**6**	**3%**
Lq, expected number in queue	**1.0343**		**7**	**2%**
L, expected number in system	**2.3586**		**8**	**2%**
Wq, expected time in queue	**0.8735**	min	**9**	**1%**
W, expected total time in system	**1.9921**	Min	**10**	**1%**
Probability that customer waits	**53%**		**11**	**0%**
			12	**0%**

Table 4: Summary of sensitivity analysis for Itakpe station

	S = 2	S = 3	S = 4
Arrival	2.089	2.089	2.089
Departure	1.787	1.787	1.787
Service utilization	58%	39%	29%
P(0)	26%	30%	31%
Ws	0.85	0.6	0.6
Ls	1.78	1.25	1.18
Prob that a customer waits	43%	13%	3%
Cost of idle server time per minute	₦2.60	₦3.00	₦3.10
cost of making provision for customers waiting for service per minute	₦1.78	₦1.25	₦1.18
Total cost	**₦4.38**	**₦4.25**	**₦4.28**

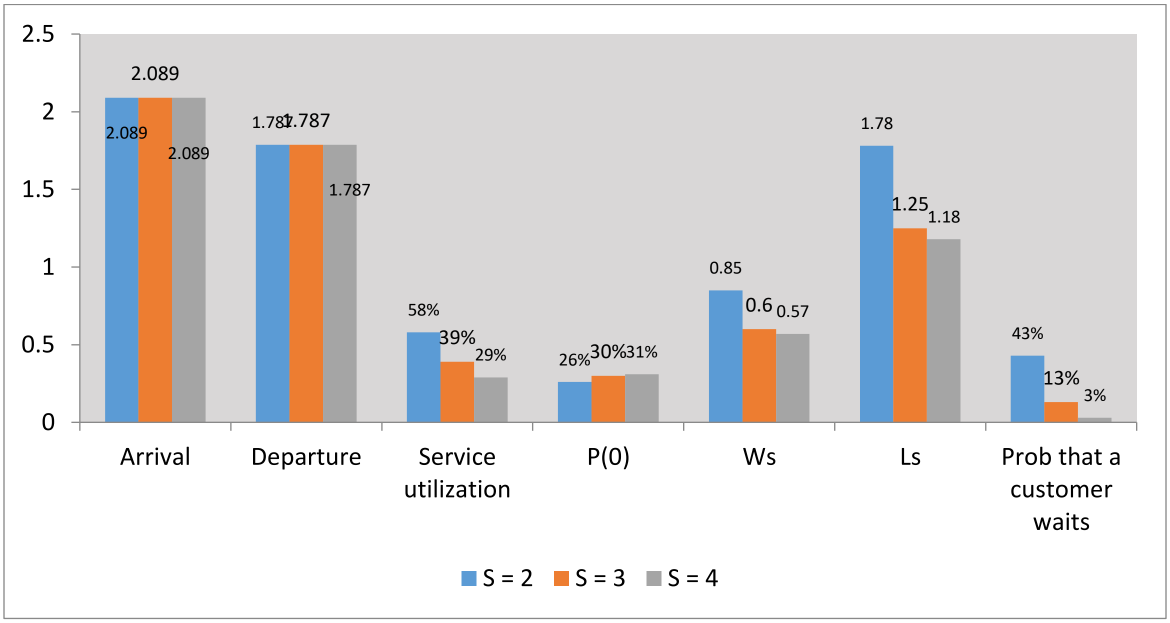

Figure 3: Summary of sensitivity analysis for Itakpe station

Table 5: Summary of sensitivity analysis for Ujevwu station

	S = 2	S = 3	S = 4
Arrival	**1.184**	**1.184**	**1.184**
Departure	**0.894**	**0.894**	**0.894**
Service utilization	66%	44%	33%
P(0)	20%	26%	26%
Ws	1.99	1.23	1.14
Ls	2.36	1.47	1.35
Prob that a customer waits	53%	18%	5%
Cost of idle server time per minute	₦2.00	₦2.60	₦2.60
cost of making provision for customers waiting for service per minute	₦2.36	₦1.47	₦1.35
Total cost	₦4.36	₦4.07	₦3.95

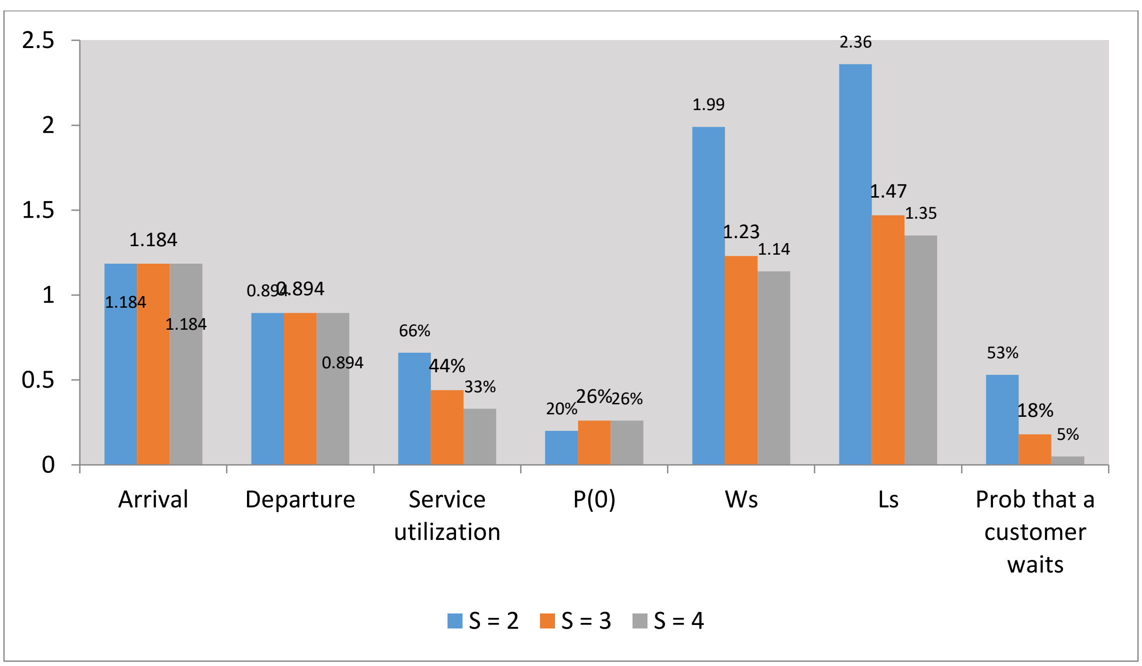

Fig 4: Summary of sensitivity analysis for Ujevwu station

6. Discussion of Results

The ticket section of Ujevwu and Itakpe was observed for the arrival and departure patterns of passengers for a period of 24 days for each station within the hours of 6 am-8 am and 1 pm-3 pm for respectively. It was observed from Table 1 that based on the data collected, the total average arrival rate between 1:00pm-3:00pm (2 hours) daily for 24 working days at the **Itakpe train station** is 250.7 passengers/day and departure rate of 214.4 passengers/day. The observed data for the study was based on 2 hours (120 minutes). Thus, for hourly observation, the average arrival rate becomes 2.089 passengers/min and average departure rate of 1.787 passengers/min. With a total of 327, 326, and 322 arrivals and 303, 294, and 299 departures, respectively, on the 24th, 23rd, and 17th day (Saturday 10/09/2022, 9/09/2022, 02/09/2022) of the observation at Itakpe station, the system was at its busiest. Also, in Table2, it shown that at the **Ujevwu train station**, there are 142.1 people arriving everyday on average between 6:00 and 8:00 AM (2 hours) and 107.3 leaving on average per day during the course of 24 working days. The study's observed data were based on two hours (120 minutes). Accordingly, the average arrival rate for one minute observation is 1.184 passengers per minute, and the average departure rate is 0.894 passengers per minute. Figure 2's presentation demonstrates a structured distribution in the system arrival and departure, it was observed that the frequency shift roughly every five days, peaking during the sixth sequence of the observation. With a total of 208 and 205 arrivals and 166 and 172 departures, respectively, on the 18th and 24th day of the study, the distribution reached its apex.

When compared to Ujevwu station, the arrival rate at Itakpe train station is higher because more people use the service, as seen by the results in tables 5. The Ujevwu station appears to be more active, however, having the larger percentage of service use (66%), even though both systems are functioning in the M/M/2 system. Additionally, Ujevwu has a lower chance of having no customers waiting for service (20%), meaning that there is a higher likelihood that at least one customer will be waiting for services there (53%) with an average wait time of two minutes per customer and an anticipated queue length of more than two (2) customers. According to all indications, the outcome reveals that Ujevwe train station is more likely to experience waiting lines.

The sensitivity analysis was also performed and from Table 4 and 5, Figure 3 and 4 using a new levels of services channels in order to further reduce the waiting line. The results showed that using three service channels will yield optimal management cost and at the same time has 30% chances of having no passengers waiting to be served and the traffic intensity will be 39% as opposed to the current 59%.

7. Conclusion

An estimation of ticket activities of two train stations have been carried out using the multi-server queuing model. It was shown that at the Itakpe train station, the average arrival rate was 2.089/min and the departure rate was 1.787 passage/min. it can be seen from Figure 1 that system was busiest on the 24th, 23rd and 17th day. In the same vein, the Ujevwu recorded an average arrival rate of 1.184/min and average departure rate of 0.894/min.

When compared to the Ujevwu station, the arrival rate at Itakpe train station is higher because more people use the service as seen in Tables 4 and 5. We analysed the traffic experienced at the two selected stations by considering the average time spent by passengers in queue and in the system, as well as the traffic intensity. The study shows how effective the use of queuing theory and its model can optimize the problem at ticket windows in the railway.

It has also been demonstrated how queuing theory can aid the reduction of excessive queues and time wasting as well as optimizing cost.

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