Power Samade distribution: its properties and application to real lifetime data

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Abstract

A new three parameter lifetime distribution has been introduced in this research work by using Gamma generalized family of distributions. The density, survival, and hazard functions of the proposed Power Samade distribution were derived and discussed. Its mathematical properties were derived. The maximum likelihood method of estimation was used to estimate its parameters. Three lifetime data sets were used to assess the performance of the proposed distribution. Our finding revealed that the Power Samade distribution suited all the data sets compared to the other competing distributions as it has a maximum value of log-likelihood and least values of statistic criteria including AIC, AICC and BIC.

Keywords: mixed distribution, Samade distribution, survival function, Moments, Order statistics

1 Introduction

Recently, different univariate lifetime probability distributions including Samade, Rayleigh, Gamma and generalized Lindley among others have been applied in statistical analysis for modelling time-to-event data. Many researchers used these distributions in different fields of applicability. For example, Rayleigh (1880) developed the Rayleigh model with increasing failure rates and applied it in Wind speed data. The origin and structural properties of the Rayleigh model have been discussed in Siddique (1962).

Many of these lifetime distributions are a mixture of exponential and gamma distributions, such as Lindley distribution (Lindley, 1958). Lindley used a mixture of exponential (θ) and gamma (2, θ) distributions to develop Lindley distribution for modelling lifetime data. Studies on the properties of the distribution have shown that it may provide a better fitting than the exponential distribution for some data sets.

Aijaz et al., (2020) introduced Hamza distribution and derived its mathematical properties. The Hamza distribution is a mixture of exponential (θ) and gamma (7, θ) distributions with combining proportion, $p = \frac{\alpha\theta^5}{120+\alpha\theta^5}$. The model was used to fit the breaking stress of carbon fibres data. The performance of the Hamza distribution has been examined and it gives an adequate fit for the data sets. Aderoju and Adeniyi (2022) introduced Power Generalized Akash distribution and extensively discussed its Properties and Applications to real-life data. The model was compared with some competing models and it performed satisfactorily in modelling lifetime data.

Most recently, a new lifetime distribution called Samade was introduced and its mathematical properties were derived by Aderoju (2021). The Samade distribution is a mixture of exponential (θ) and gamma (4, θ) distributions, which was expressed as

$$f(x|\alpha,\theta) = pg_1(x;\theta) + (1-p)g_2(x;4,\theta),$$

where

$$p = \frac{\theta^4}{\theta^4 + 6\alpha}, \quad g_1(x;\theta) = \theta e^{-\theta x} \quad and \quad g_2(x;4,\theta) = \frac{\theta^4 x^3 e^{-\theta x}}{6}$$

Hence, (1) was defined as:

$$f(x|\alpha,\theta) = \frac{\theta^4}{\theta^4 + 6\alpha}(\theta + \alpha x^3)e^{-\theta x}, \qquad x > 0$$

where

$\theta > 0$ and $\alpha > 0$.

Note that when $\alpha = 1$ and $\alpha = 0$, $f(x|\alpha,\theta)$ reduces to the one-parameter Pranav distribution and exponential distribution, respectively.

The corresponding cumulative distribution function (CDF) of (1) is:

$$F(x|\alpha,\theta) = 1 - \frac{\left(\theta^4 + \alpha\left(6 + x\theta\left(6 + x\theta(3 + x\theta)\right)\right)\right)}{6\alpha + \theta^4} e^{-x\theta}, \quad x > 0 \qquad (2)$$

where

$\theta > 0$ and $\alpha > 0$.

The authors compared the goodness of fit of the distribution with some selected two-parameter lifetime distributions and found it to be better than the existing models.

The aim of this paper is to develop a generalization of Samade distribution. We introduce the proposed distribution in Section 2 and study its mathematical properties in section 3. The corresponding Maximum Likelihood estimates of the parameters are discussed in Section 4. In Section 5, we discuss its application to real-life examples and we present concluding remarks in Section 6.

2 Materials and Methods

In this section, we present the new three-parameter distribution called Power Samade distribution (PSD) by introducing a shape parameter to the Samade probability density function. Aderoju (2021) studied the Samade distribution in great detail with its application. However, distribution extended from a theoretical or applied point of view. So, to obtain a more flexible distribution, we introduce here a new extension of the Samade distribution by considering the power transformation $X = T^{\omega^{-1}}$. The PDF of the X can be readily obtained as

$$f(x|\alpha,\theta,\omega) = pg_1(x;\theta) + (1-p)g_2(x;4,\theta),$$

where

$$p = \frac{\theta^4}{\theta^4 + 6\alpha},$$

$$g_1(x;\theta) = \omega\theta x^{\omega-1} e^{-\theta x^{\omega}}, \qquad x > 0$$

$$g_2(x;4,\theta) = \frac{\omega\theta^4 x^{3\omega-1} e^{-\theta x^{\omega}}}{6}, \qquad x > 0$$

$$\therefore f(x|\alpha,\theta,\omega) = \begin{cases} \dfrac{\omega\theta^4 x^{\omega-1}}{(\theta^4 + 6\alpha)}(\theta + \alpha x^{3\omega}) e^{-\theta x^{\omega}}, & for\ x,\alpha,\theta,\omega > 0 \\ 0, & elsewhere \end{cases} \qquad (3)$$

The Power Samade distribution is a two-component mixture of Weibull distribution (with shape ω and scale θ), and a generalized gamma distribution (with shape parameters 4, ω and scale θ), with mixing proportion $p = \frac{\theta^4}{\theta^4+6\alpha}$.

$$f(0|\alpha,\theta,\omega) = \begin{cases} \infty, & if\ \omega < 1 \\ \dfrac{\omega\theta^5}{\theta^4 + 6\alpha}, & if\ \omega = 1 \\ 0, & if\ \omega > 1 \end{cases}$$

and

$$f(\infty|\alpha,\theta,\omega) = 0.$$

The probability density function of the newly developed three-parameter lifetime distribution can be expressed as (3):

Note that the shape characteristics of the pdf *f(x)* in (3) of the $PS(\alpha,\theta,\omega)$ distribution. The behaviour of $f(x|\alpha,\theta,\omega)$ at $x = 0$ and $x = \infty$, respectively, are given by

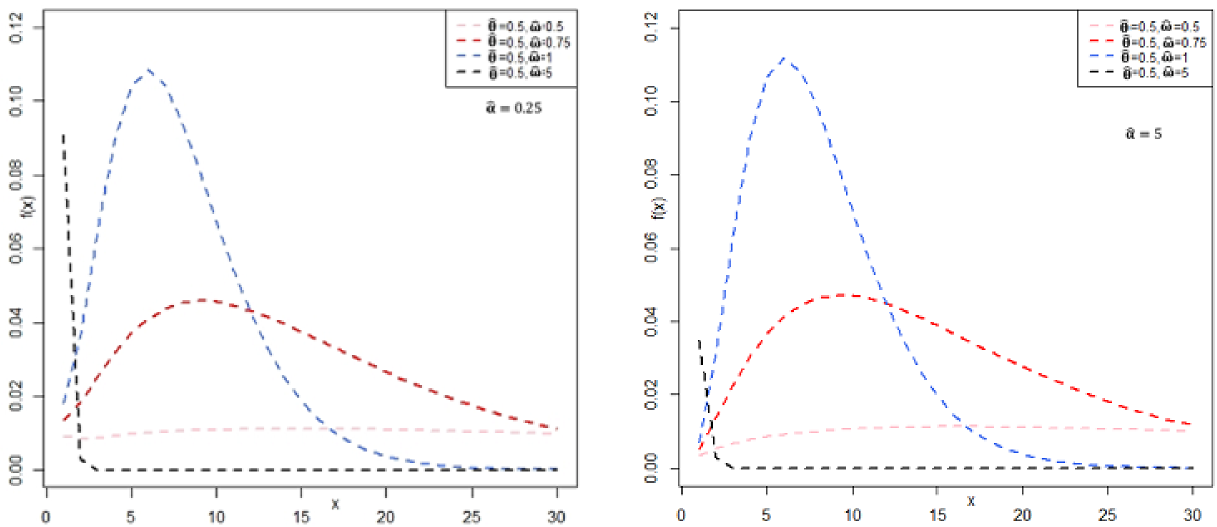

Figure 1: Plots of the probability density function of Power Samade distribution for different parameter values

The corresponding cumulative distribution function (cdf) of (3) is obtained as:

$$F(x) = \int_0^x f(t|\alpha,\theta,\omega)\,dt$$

$$= \int_0^x \frac{\omega\theta^4}{\theta^4+6\alpha} t^{\omega-1}(\theta+\alpha t^{3\omega})e^{-\theta t^{\omega}}dt$$

$$F(x) = 1 - \left[\frac{\theta^4+\alpha\left(6+\theta x^{\omega}\left(6+\theta x^{\omega}(3+\theta x^{\omega})\right)\right)}{6\alpha+\theta^4}\right]e^{-\theta x^{\omega}} \quad (4)$$

The graphical representation of the density function and cdf of the PSD for some fixed values of the parameters are presented below.

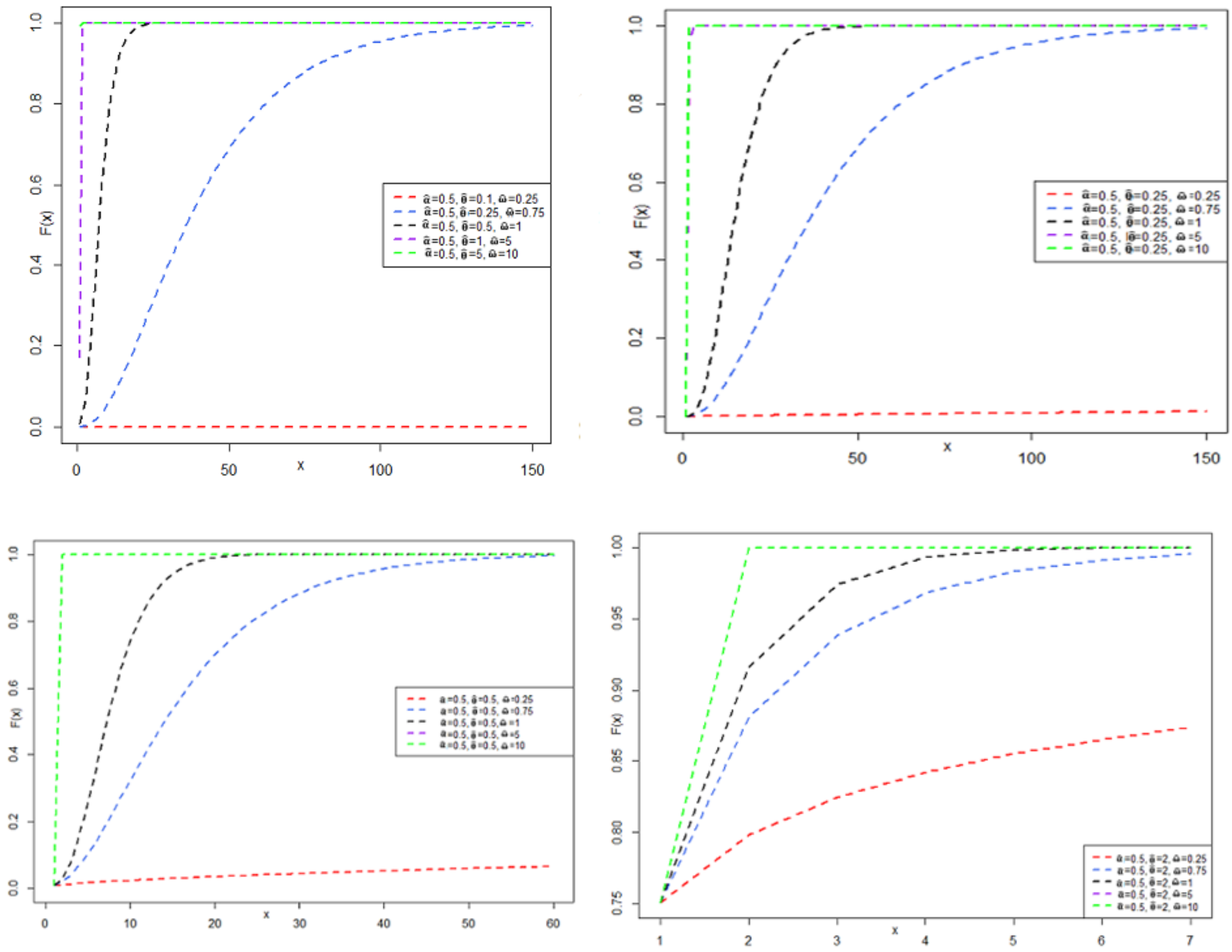

Figure 2: Plots of the CDF function of Power Samade distribution for different parameter values

3 Statistical Properties of Power Samade distribution

We present some of the Statistical properties of the Power Samade distribution in this section. The properties include the moments, reliability analysis and order statistics.

3.1 Moments

Suppose a random variable X follows power Samade distribution, that is $X \sim PS(\alpha, \theta, \omega)$, then, the r^{th} order moment about the origin is given by:

$$E(X^r) = \mu_r = \int_0^{\infty} x^r f(x)dx$$

$$= \int_0^{\infty} x^r \frac{\omega\theta^4 x^{\omega-1}}{(\theta^4 + 6\alpha)}(\theta + \alpha x^{3\omega})e^{-\theta x^{\omega}}dx \qquad (5)$$

Hence, the first four moments are obtained as:

$$\mu_1 = \frac{\alpha\omega\Gamma\left(4 + \frac{1}{\omega}\right) + \theta^4\Gamma\left(\frac{1}{\omega}\right)}{\theta^{1/\omega}(\theta^4 + 6\alpha)\omega}$$

$$\mu_2 = \frac{\alpha\Gamma\left(4 + \frac{2}{\omega}\right) + \theta^4\Gamma\left(\frac{2+\omega}{\omega}\right)}{\theta^{2/\omega}(\theta^4 + 6\alpha)}$$

$$\mu_3 = \frac{\alpha\Gamma\left(4 + \frac{3}{\omega}\right) + \theta^4\Gamma\left(\frac{3+\omega}{\omega}\right)}{\theta^{3/\omega}(\theta^4 + 6\alpha)}$$

$$\mu_4 = \frac{\alpha\Gamma\left(4 + \frac{4}{\omega}\right) + \theta^4\Gamma\left(\frac{4+\omega}{\omega}\right)}{\theta^{4/\omega}(\theta^4 + 6\alpha)}$$

Note that, the variance (σ^2) of the random variable X can be obtained as:

$$\sigma^2 = E\left(X^2\right) - \left[E\left(X^1\right)\right]^2 = \mu_2 - [\mu_1]^2$$

$$\therefore\ \sigma^2 = \frac{\left[\left(\theta^4 + 6\alpha\right)\left(\alpha\Gamma\left(4 + \frac{2}{\omega}\right) + \theta^4\Gamma\left(\frac{2+\omega}{\omega}\right)\right) - \frac{\left(\alpha\omega\Gamma\left(4 + \frac{1}{\omega}\right) + \theta^4\Gamma\left(\frac{1}{\omega}\right)\right)^2}{\omega^2}\right]}{\theta^{2/\omega}\left(\theta^4 + 6\alpha\right)^2}$$

The corresponding coefficient of variation (CV) and the index of dispersion (γ) of PSD are obtained as:

$$CV = \frac{\sigma}{\mu_1}$$

$$= \frac{\sqrt{\left(\theta^4 + 6\alpha\right)\left(\alpha\Gamma\left(4 + \frac{2}{\omega}\right) + \theta^4\Gamma\left(\frac{2+\omega}{\omega}\right)\right) - \frac{\left(\alpha\omega\Gamma\left(4 + \frac{1}{\omega}\right) + \theta^4\Gamma\left(\frac{1}{\omega}\right)\right)^2}{\omega^2}}}{\theta^{1/\omega}\left(\theta^4 + 6\alpha\right)\left(\alpha\omega\Gamma\left(4 + \frac{1}{\omega}\right) + \theta^4\Gamma\left(\frac{1}{\omega}\right)\right)}$$

$$\gamma = \frac{\sigma^2}{\mu_1}$$

$$\gamma = \frac{\left(\theta^4 + 6\alpha\right)\left(\alpha\Gamma\left(4 + \frac{2}{\omega}\right) + \theta^4\Gamma\left(\frac{2+\omega}{\omega}\right)\right) - \frac{\left(\alpha\omega\Gamma\left(4 + \frac{1}{\omega}\right) + \theta^4\Gamma\left(\frac{1}{\omega}\right)\right)^2}{\omega^2}}{\theta^{1/\omega}\left(\theta^4 + 6\alpha\right)\left(\alpha\omega\Gamma\left(4 + \frac{1}{\omega}\right) + \theta^4\Gamma\left(\frac{1}{\omega}\right)\right)}$$

3.4 Reliability Analysis

The reliability characteristics of any given pdf are always considered based on the survival function and the hazard rate function of the distribution. Therefore, the survival function, $S(x)$, and the hazard rate function, $h(x)$, were derived as shown below:

$$S(x) = 1 - F(x)$$

$$S(x) = \left[\frac{(\theta^4 + \alpha(6 + 6x^{\omega}\theta + 3x^{2\omega}\theta^2 + x^{3\omega}\theta^3))}{6\alpha + \theta^4}\right] e^{-x^{\omega}\theta}$$

3.4.1 Survival Function

The survival function is generally defined as the probability that an item does not fail prior to some time. It is expressed as:

3.4.2 Hazard rate function

The hazard rate function can be expressed as the conditional probability of failure, given that it has survived to the time. It is given as:

$$h(x) = \frac{f(x)}{S(x)}$$

$$h(x) = \frac{\theta^4(\alpha x^{3\omega} + \theta)\omega x^{\omega-1}}{\theta^4 + \alpha(6 + 6x^{\omega}\theta + 3x^{2\omega}\theta^2 + x^{3\omega}\theta^3)}$$

Figures 3 and 4 represent the graph of the survival function and hazard rate function of the Power Samade distribution, respectively, for varying values of the parameters $\hat{\alpha}, \hat{\theta}$ and $\hat{\omega}$.

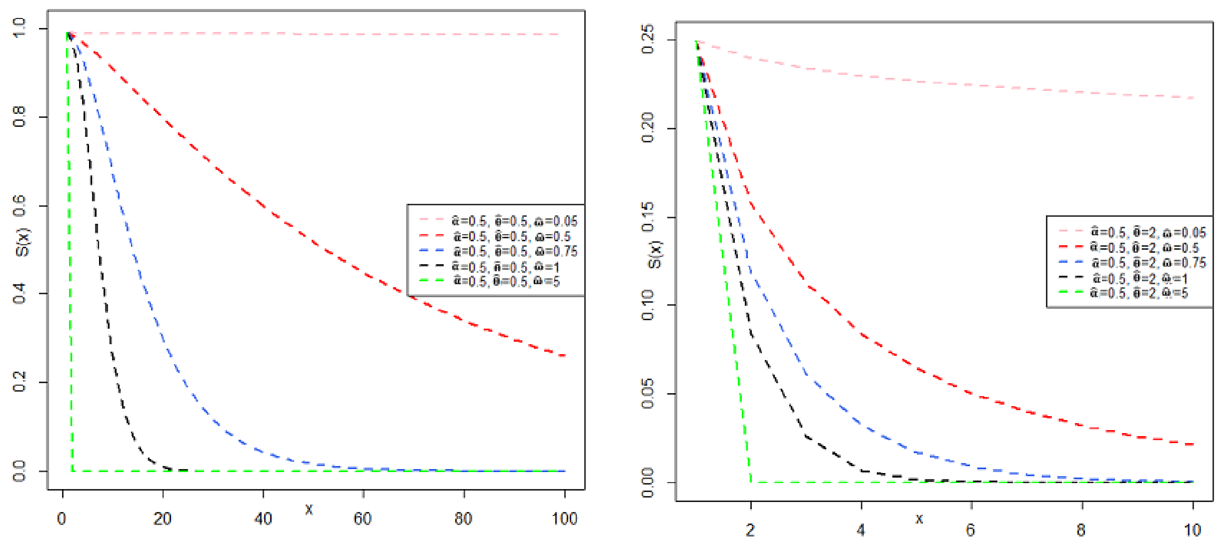

Figure 3: Plots of the survival function of Power Samade distribution for different parameter values

Plotting the $S(x)$ of the $PS(\alpha, \theta, \omega)$ for different values of the parameters (α, θ, ω) in Figure 3 show that, the shapes of $S(x)$ are decreasing for all selected values of the shape parameters (α, ω) and the scale parameter (θ); this also shows how flexible the behaviour of the $S(x)$.

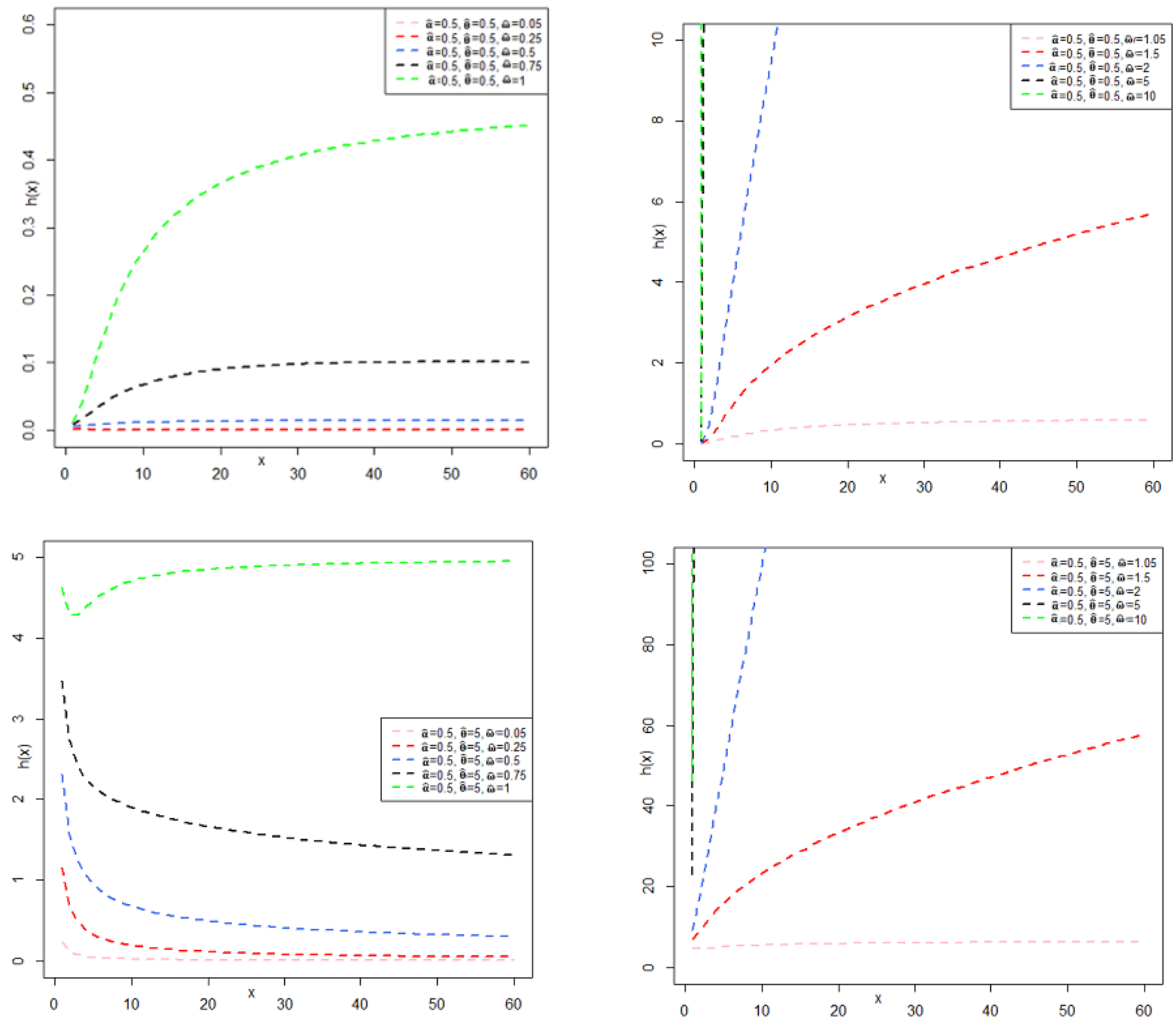

Figure 4: Plots of the Hazard rate function of Power Samade distribution for different parameter values

The graphs of the hazard rate function of the PSD for different values of the parameters are given in Figure 4. Obviously, the model exhibits both monotones increasing and decreasing failure characteristics. It decreases monotonically when $\omega < 1$ and increases monotonically when $\omega \geq 1$.

3.3 Order Statistics

Order statistics gives one of the popular fundamental tools for obtaining inference related to reliability data. The largest order and the smallest are denoted as $X_n = \max(X_1, X_2, \ldots, X_n)$ and $X_1 = \min(X_1, X_2, \ldots, X_n)$ where n is the sample size. Suppose $X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(n)}$ are order observations of $X_1, X_2, \ldots, X_n$ taken from the studied distribution then the density of the k^{th} order statistic $X_{(k)}$ can be expressed as:

$$f_{X(k)}(x) = \frac{n!}{(k-1)!\,(n-k)!} f(x)[F(x)]^{k-1}[1-F(x)]^{n-k} \qquad (7)$$

$$= \frac{n!\,\omega\theta^4 x^{\omega-1}}{\left(6\alpha+\theta^4\right)(k-1)!\,(n-k)!}\left(\theta+\alpha x^{3\omega}\right)e^{-\theta x^{\omega}}\left[1-\left[\frac{\theta^4+\alpha\left(6+\theta x^{\omega}(6+\theta x^{\omega}(3+\theta x^{\omega}))\right)}{6\alpha+\theta^4}\right]e^{-\theta x^{\omega}}\right]^{k-1}$$

$$\times\left[\left[\frac{(\theta^4+\alpha(6+6x^{\omega}\theta+3x^{2\omega}\theta^2+x^{3\omega}\theta^3))}{6\alpha+\theta^4}\right]e^{-x^{\omega}\theta}\right]^{n-k}$$

The first and n^{th} orders are:

$$f_{X(1)}(x) = \frac{n!\,\omega\theta^4 x^{\omega-1}}{\left(6\alpha+\theta^4\right)(k-1)!\,(n-k)!}\left(\theta+\alpha x^{3\omega}\right)e^{-\theta x^{\omega}}\left[\left[\frac{(\theta^4+\alpha(6+6x^{\omega}\theta+3x^{2\omega}\theta^2+x^{3\omega}\theta^3))}{6\alpha+\theta^4}\right]e^{-x^{\omega}\theta}\right]^{n-1}$$

$$f_{X(n)}(x) = \frac{n!\,\omega\theta^4 x^{\omega-1}}{\left(6\alpha+\theta^4\right)(k-1)!\,(n-k)!}\left(\theta+\alpha x^{3\omega}\right)e^{-\theta x^{\omega}}\left[1-\left[\frac{\theta^4+\alpha\left(6+\theta x^{\omega}(6+\theta x^{\omega}(3+\theta x^{\omega}))\right)}{6\alpha+\theta^4}\right]e^{-\theta x^{\omega}}\right]^{n-1}$$

4 Maximum Likelihood Estimation

Suppose $X_1, X_2, X_3, \ldots, X_n$ is a random sample of size *n* from the PSD, the maximum likelihood function of parameters can be written as

$$L(\alpha,\theta,\omega) = \prod_{i=1}^{n}\frac{\omega\theta^4 x^{\omega-1}}{\left(\theta^4+6\alpha\right)}\left(\theta+\alpha x^{3\omega}\right)e^{-\theta x^{\omega}},$$

and the log-likelihood function is

$$lnL = \ell = lnL(\alpha,\theta,\omega)$$
$$= nln(\omega)+4nln(\theta)+(\omega-1)\sum_{i=1}^{n}lnx_i - nln\left(\theta^4+6\alpha\right)+\sum_{i=1}^{n}\ln\left(\theta+\alpha x^{3\omega}\right) - \theta\sum_{i=1}^{n}x_i^{\omega}$$

Hence,

$$\frac{\partial lnL}{\partial\alpha} = -\frac{6n}{(\theta^4+6\alpha)}+\sum_{i=1}^{n}x^{3\omega}\left(\theta+\alpha x^{3\omega}\right)^{-1} = 0 \qquad (10)$$

$$\frac{\partial lnL}{\partial\theta} = \frac{4n}{\theta}-\frac{4n\theta^3}{\theta^4+6\alpha}+\sum_{i=1}^{n}\left(\theta+\alpha x^{3\omega}\right)^{-1}-\sum_{i=1}^{n}x_i^{\omega} = 0 \qquad (11)$$

$$\frac{\partial lnL}{\partial \omega} = \frac{n}{\omega} + \sum_{i=1}^{n} \ln(x_i) + \sum_{i=1}^{n} \frac{3\alpha {x_i}^{3\omega} \ln(x_i)}{\theta + \alpha x^{3\omega}} - \theta \sum_{i=1}^{n} x_i^{\omega} \ln(x_i) = 0 \quad (12)$$

Differentiating the $logL(\alpha, \theta, \omega)$ partially with respect to associated parameters and equate the zero as shown in (10), (11) and (12) should provide solution algebraically; the Maximum Likelihood Estimates (MLEs), $\hat{\alpha},\ \hat{\theta}\ and\ \hat{\omega}$, of α, θ and ω are solutions of the equations. Obviously, it is very difficult to solve the system of nonlinear equations, hence, we computed the MLEs numerically using the *nloptr* package and *bobyqa* function in R software (R Core Team, 2022).

5 Application

In order to compare the Power Samade distribution with some other distributions, we consider the criteria like Bayesian information criterion *(BIC)*, Akaike Information Criterion *(AIC)*, Akaike Information Criterion Corrected *(AICC)* and *-2logL*. The better distribution is which corresponds to lower values of *AIC, BIC, AICC* and *-2logL*. For calculating *AIC, BIC, AICC* and *-2logL* can be obtained by using the formulas as follows:

$$AIC = 2K - 2logL,$$

$$BIC = klogn - 2logL,$$

and

$$AICC = AIC + \frac{2k(k+1)}{n-k-1}$$

where *k* is the number of parameters, *n* is the sample size and *-2logL* is the maximized value of log-likelihood function and are showed in table 1 and table 2.

Table 1: Distributions considered

Name of distributions	Probability density functions	Introducers / Authors
Samade distribution (SD)	$\frac{\theta^4}{\theta^4 + 6\alpha}(\theta + \alpha x^3)e^{-\theta x}$	Aderoju (2021)
Hamza Distribution (HD)	$f(x) = \frac{\theta^6}{(120 + \alpha\theta^5)}\left(\alpha + \frac{\theta}{6}x^6\right)e^{-\theta x}$	Aijaz et al., (2020)
Power Pranav Distribution (PPD)	$f(x) = \frac{\omega\theta^4 x^{\omega-1}}{(2 + \theta^4)}(\theta + x^{3\omega})e^{-\theta x^{\omega}}$	Shukla (2019)
Power Quasi Lindley Distribution (PQLD)	$f(x) = \frac{\omega\theta x^{\omega-1}}{(1 + \alpha)}(\alpha + \theta x^{\omega})e^{-\theta x^{\omega}}$	Alkarni (2015)
Power Hamza Distribution (PHD)	$f(x) = \frac{\omega\theta^6 x^{\omega-1}}{(120 + \alpha\theta^5)}\left(\alpha + \frac{\theta}{6}x^{6\omega}\right)e^{-\theta x^{\omega}}$	Aderoju & Jolayemi (2022)

DATA SET 1: The data set represents the strength of 1.5cm glass fibers measured at the National Physical Laboratory, England. Unfortunately, the units of measurements are not given in the paper, and they are taken from Smith and Naylor (1987)

0.55, 0.93, 1.25, 1.36, 1.49, 1.52, 1.58, 1.61, 1.64, 1.68, 1.73, 1.81, 2.00, 0.74, 1.04, 1.27, 1.39, 1 .49, 1.53, 1.59, 1.61, 1.66, 1.68, 1.76, 1.82, 2.01, 0.77, 1.11, 1.28, 1.42, 1.50, 1.54, 1.60, 1.62, 1. 66, 1.69, 1.76, 1.84, 2.24, 0.81, 1.13, 1.29, 1.48, 1.50, 1.55, 1.61, 1.62, 1.66, 1.70, 1.77, 1.84, 0.8 4, 1.24, 1.30, 1.48, 1.51, 1.55, 1.61, 1.63, 1.67, 1.70, 1.78, 1.89

DATA SET 2: The second data set represents the remission times (in months) of a random sample of 128 bladder cancer patients reported in Lee and Wang (2003). See the table below.

0.08, 2.09, 3.48, 4.87, 6.94, 8.66, 13.11, 23.63, 0.20, 2.23, 3.52,4.98, 6.97, 9.02, 13.29, 0.40, 2.26, 3.57, 5.06, 7.09, 9.22, 13.80,25.74, 0.50, 2.46, 3.64, 5.09, 7.26, 9.47, 14.24, 25.82, 0.51, 2.54,3.70, 5.17, 7.28, 9.74, 14.76, 26.31, 0.81, 2.62, 3.82, 5.32, 7.32,10.06, 14.77, 32.15, 2.64, 3.88, 5.32, 7.39, 10.34, 14.83, 34.26, 0.90,2.69, 4.18, 5.34, 7.59, 10.66, 15.96, 36.66, 1.05, 2.69, 4.23, 5.41,7.62, 10.75, 16.62, 43.01, 1.19, 2.75, 4.26, 5.41, 7.63, 17.12, 46.12,1.26, 2.83, 4.33, 5.49, 7.66, 11.25, 17.14, 79.05, 1.35, 2.87, 5.62,7.87, 11.64, 17.36, 1.40, 3.02, 4.34, 5.71, 7.93, 11.79, 18.10, 1.46,4.40, 5.85, 8.26, 11.98, 19.13, 1.76, 3.25, 4.50, 6.25, 8.37, 12.02,2.02, 3.31, 4.51, 6.54, 8.53, 12.03, 20.28, 2.02, 3.36, 6.76, 12.07,21.73, 2.07, 3.36, 6.93, 8.65, 12.63, 22.69

EXAMPLE 3: This data set consists of 72 exceedances of flood peaks (in m3/s) of the Wheaton river near Carcross in Yukon Territory, Canada for the years 1958-1984. This data was first used by Choulakian and Stephens (2001) to examine the applicability of the generalized Pareto distribution and also was reported in Akinsete et al. (2008).

1.7, 2.2, 14.4, 1.1, 0.4, 20.6, 5.3, 0.7, 1.9, 13.0, 12.0, 9.3, 1.4, 18.7, 8.5, 25.5, 11.6, 14.1, 22.1, 1. 1, 2.5, 27.0, 14.4, 1.7, 37.6, 0.6, 2.2, 39.0, 0.3, 15.0, 11.0, 7.3, 22.9, 1.7, 0.1, 1.1, 0.6, 9.0, 1.7, 7. 0, 20.1, 0.4, 2.8, 14.1, 9.9, 10.4, 10.7, 30.0, 3.6, 5.6, 30.8, 13.3, 4.2, 25.5, 3.4, 11.9, 21.5, 27.6, 3 6.4, 2.7, 64.0, 1.5, 2.5, 27.4, 1.0, 27.1, 20.2, 16.8, 5.3, 9.7, 27.5, 2.5

Table 2: Parameter estimations and goodness of fit test statistics for data set 1

Dataset	**Models**	**Parameter estimates**	**-2logL**	**AIC**	**AICC**	**BIC**	**Rank**
Data 1 n=63	PSD	$\hat{\alpha} = 0.0583$ $\hat{\theta} = 0.5361$ $\hat{\omega} = 3.9901$	22.8199	28.8199	29.2267	31.1062	1
	SD	$\hat{\alpha} = 7499.1$ $\hat{\theta} = 2.6518$	93.7870	97.7870	97.9870	102.0733	5
	PHD	$\hat{\alpha} = 3.1302$ $\hat{\theta} = 1.4452$ $\hat{\omega} = 3.1962$	23.6928	29.6928	30.0996	31.9791	2
	HD	$\hat{\alpha} = 2.37 \times 10^6$ $\hat{\theta} = 0.6649$	177.7098	181.7098	181.9098	185.9961	6
	PPD	$\hat{\alpha} = 3.2042$ $\hat{\theta} = 0.8776$	26.6161	30.6161	30.8161	34.9024	3
	PQLD	$\hat{\alpha} = 0.5347$ $\hat{\theta} = 0.1559$ $\hat{\omega} = 4.9439$	28.4214	34.4214	34.8281	36.7076	4
	PSD	$\hat{\alpha} = 56.7073$ $\hat{\theta} = 1.3938$ $\hat{\omega} = 0.5178$	821.395	827.395	827.5886	831.0991	1
	SD	$\hat{\alpha} = 0.0015$ $\hat{\theta} = 0.2898$	859.975	863.975	864.071	859.975	5
	PHD	$\hat{\alpha} = 0.0022$ $\hat{\theta} = 3.2634$ $\hat{\omega} = 0.3869$	822.025	828.0249	828.026	831.729	2

Data 2 n=128	HD	$\hat{\alpha} = 5.49 \times 10^5$ $\hat{\theta} = 0.2399$	860.994	864.9941	869.5999	870.6982	6
	PPD	$\hat{\alpha} = 0.6397$ $\hat{\theta} = 0.9564$	831.374	835.374	835.5675	841.078	4
	PQLD	$\hat{\alpha} = 865.50$ $\hat{\theta} = 0.0939$ $\hat{\omega} = 1.0479$	828.1738	834.1738	834.3674	837.8779	3
Data 3 n=72	PSD	$\hat{\alpha} = 2.97 \times 10^{-3}$ $\hat{\theta} = 0.3542$ $\hat{\omega} = 0.8261$	498.8676	504.8676	505.2206	507.421	1
	SD	$\hat{\alpha} = 0.0011$ $\hat{\theta} = 0.2507$	510.5072	514.5071	514.681	519.0605	5
	PHD	$\hat{\alpha} = 8425.73$ $\hat{\theta} = 0.4248$ $\hat{\omega} = 0.9155$	500.962	506.9621	507.315	509.5154	2
	HD	$\hat{\alpha} = 45730.2$ $\hat{\theta} = 0.3124$	501.914	505.9141	506.088	510.4674	3
	PPD	$\hat{\alpha} = 0.5371$ $\hat{\theta} = 1.0491$	510.9752	514.9753	515,1492	519.5286	6
	PQLD	$\hat{\alpha} = 2.5365$ $\hat{\theta} = 0.1536$ $\hat{\omega} = 0.8705$	502.7304	508.7303	509.0833	511.2837	4

6 Conclusion

An extension of Samade distribution was introduced based on the mixture of Weibull and generalized gamma distributions. The shape of density function of the extended distribution exhibited the pattern of symmetric and right-skewed for some parameters values. Plots of hazard function of the proposed Power Samade distribution could be increasing, decreasing and constant rates. The parameters of the distribution were obtained using maximum likelihood estimation approach. An application to three data sets were used to investigate the performance of the proposed distribution in comparison to other competing models. Our finding revealed that, the proposed Power Samade distribution has maximum value of log-likelihood and minimum values of *AIC*, *AICC* and BIC criteria for the data sets considered. We hope that, the proposed Power Samade distribution could always be considered in comparison to other recent models.

Conflicts of Interest

The authors declare that they have no competing interests.

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