Evaluation of Recruitment Cost in Manpower Planning Using Dynamic Programming Model

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Abstract

In this paper, we present a manpower planning model using dynamic programming approach. The model is first formulated as a linear programming model and then converted to a dynamic programming model by using the duality theorem which makes it possible for computer implementation. The optimal solution to the proposed DP model in LP form in when applied to a given numerical example using Program Full-Simplex, the results reveal that out of the ten basic variables in the optimal tableau, seven of them which are surplus are non–decision variables while the remaining three are decision variables that contribute to the objective function value. It was also observed that the proposed dynamic programming model in linear programming form has a computational advantage of quick and accurate solutions over earlier existing models due to its computer implementation.

Keywords: dynamic programming, manpower, recruitment, overstaffing

Introduction

Manpower as stated in Bontis et al (1999) is human factor which includes intelligence, skills and expertise as regard to production of goods and services in any organization. Assessing future manpower requirement in terms of number and competence has remained a challenge to human resources managers in both private and public organizations. Hence, Gregoriades (2000) remarked that manpower is the most valuable crucial and unpredictable assets in any organization and that failure to put the right people in the right position at the right time can lead to business failure. According to Kathry Valier ((2023), hiring and retaining the right employees is one of the biggest challenges to employers. Ranslam (2022) remarked that understaffing and overstaffing will continue to be a challenge as long as there is business to run. Sources of manpower supply in any organization are both external and internal. External manpower supply sources involve recruiting staff from outside the establishment while internal manpower supply sources involves transfer, redeployment and promotion within an establishment, as stated in Ogumeyo (2010).

Yan and Chen (2008), state that the goal of manpower planning involves taking into account various environmental factors of an industry sure as recruitment, promotion, retirement, resignation etc. to sure future demand and supply in the workforce coincide optimally. Thus, a manpower planning model which identifies the dynamics of manpower in terms of recruitment and retrenchment /resignation in an uncertain environment is developed in Mutingi and Mbohwa (2012) which is an extension of Parker's (1996) dynamic manpower planning model which includes the flow of manpower through the three stages: recruitment, promotion and retirement.

Manpower planning models is classified into three major groups in operations research: optimization; Markovian models and computer simulation, Cai et al (2010). The main techniques used in the optimization models are linear programming, integer programming, goal programming and dynamic programming. Dynamic programming (DP) is a mathematical technique in which a given problem involving a series of interrelated decisions is divided into sub-problems called stages whereby lower dimensional optimization takes place, Wagner (2001). Rahela (2015*) analyzed data of higher learning institution using Markov chain with the objective of designing a manpower* planning model which project future employments in a university faculty. Wan-yin and Shou, (2015) studied gray Markov model in human resource internal supply forecast. Verbeken and Guerry (2021) developed a discrete time hybrid semi- Markov model in manpower planning with the aim of reducing model complexity in terms of staff to be recruited and retained in a manpower system. A conceptual framework of a simulation- based manpower panning model is studied by Biruk et al (2022) to determining recruitment patterns in an uncertain environment for construction companies. This allows the evaluation of the effects of maintaining different levels of employment of the workforce.

Ezugwu and Ologun (2017) developed a predictive manpower planning model to determine the proportion of academic staff that should be recruited, promoted and withdrawn from the various grades and to forecast the academic staff of the university in the next five years using Markov chain model.

A multinomial hidden Markov model for hierarchical manpower system is studied in Udom and Ebedoro (2021).

Some examples of DP models for manpower planning in operations research literature include: the optimal recruitment and transition strategies for manpower system using dynamic programming approach developed in Mehlmann (1980), Rao's (1990) dynamic programming model in a linear programming form in which the cost structure consists of recruitment and overstaffing while Nirmala and Jeeva (2012) extended the work of Rao (1990) by including promotion cost factor to Rao's (1990) dynamic programming model in a linear programming form. The problem is to minimize the total manpower cost within a given period of time, subject to the constraint that all recruitments and promotions must be met on time to avoid understaffing.

The proposed DP model in linear programming is an extension of those of Rao (1990) and Nirmala and Jeeva (2012) manpower planning models and is meant to correct identified shortcomings in them for better application to real life situations.

Model Assumptions and Notations

We first state the assumptions and notations which are contained in Rao (1990) as follows:

Assumptions

The following are the assumptions of the DP model in LP form for manpower planning based on recruitment.

(a) The recruitment size is known and fixed.

(b) Recruitment at a particular grade is considered.

(c) Recruitment and overstaffing costs are known.

(d) Understaffing is not allowed.

Notations

R_j = requirement in period j

k_j = fixed recruitment cost in period j

l_j = cost of overstaffing per recruited staff per period

p_j = number of people recruited in period j

y_j = number of people recruited in an earlier period for the requirement of period j

v = recruitment cost per recruited employee in period j.

n = Number of periods

$n-r$ = the general period which has r more periods ahead of it

$d_{j,n-r}$ = the decision to recruit for the first j periods at period $n-r$

$c_{j,n-r}$ = the cost of recruiting for the first j periods at period $n-r$

g_j = number of people promoted in period j from grade 1 to grade 2

u_j = cost of promotion per staff in period j from grade 1 to grade 2.

y_{jk} = is the number of people recruited earlier for period j at grade k

R_{jk} = is the requirement in period j for grade k

$Q_j =$ number of people required for promotion in period j

Formulation of the Proposed DP Model in LP form

Let n be the number of stages or periods in which recruitment is planned for in a given establishment. Since the manpower recruitment requirements and fixed recruitment costs vary from period to period, the overstaffing cost per recruited staff per period (which is denoted by l_j) also varies from period to period.

In Rao (1990), manpower planning model, we need to satisfy all requirements on time, so that understaffing is prohibited. The model is founded on the assumption that the number of staff required in period j (R_j) can be estimated (with known unit overstaffing cost), which can be used to compute the number of staff that can be recruited in period j (y_j). The model uses dynamic programming approach and periodic fixed recruitment cost in such a way that understaffing is not allowed.

Rao's DP model in LP form is thus, stated as follows:

$$\left.\begin{aligned} &Minimize\ \ z = \sum_{j=1}^{n}\left[k_j\delta p_j + v_j p_j + l_j y_j\right] \quad (total\ \text{cost}\ \ of\ \ recruitment) \\ &s.t. \\ &\sum_{j=1}^{i} y_j = \sum_{j=1}^{i} R_j, \quad i = 1,\ 2, \cdots, n \cdot \qquad (constraints\ of\ the\ DP\ model\ 2) \\ &y_i \geq 0\ \ ,\ \ j = 1(1)n \qquad (nonnegativity\ \ constraints) \end{aligned}\right\} \qquad (1)$$

$$\delta p_j = \begin{cases} 1 & if \quad p_j > 0 \\ 0 & if \quad p_j = 0 \end{cases}$$

$l_j y_j$ is the overstaffing cost.

We take $y_0 = y_n =$ without loss of generality.

Thus, the periodic requirements (R_j), fixed recruitment costs and unit overstaffing costs for total recruitment cost can be tabulated as in Table 1.1

Table 1.1: Periodic data

Period	No. of staff required (R_j)	Fixed recruitment cost k_j (₦)	Unit overstaffing cost l_j (₦)
1 2 3	R_1 R_2	k_1 k_2	l_1 l_2

$\vdots$ n	R_3 $\vdots$ R_n	k_3 $\vdots$ k_n	l_3 $\vdots$ l_n

Table 1.1 is used in such a way that the cost of each sub-decision takes earlier decisions into consideration according to the principle of optimality in dynamic programming.

Similarly, Nirmala and Jeeva [12] DP model in LP form is stated as follows:

$$\left.\begin{aligned} &Minimize\ z = \sum_{j=1}^{n}\left[k_j \delta p_j + v_j p_j + u_j g_j + l_j y_j\right], \\ &subject\ \ to \sum_{j-1}^{i} y_{jk} = \sum_{j-1}^{i} R_{jk},\ i = 1,\ 2, \cdots, n;\ \ k = 1, 2 \\ &\qquad\qquad \sum_{j=1}^{i} g_j = \sum_{j=1}^{i} Q_j,\ i = 1,\ 2, \cdots, n \end{aligned}\right\} \qquad (2)$$

The type of linear constraints in systems (1) and (2) are typical of dynamic systems hence they are DP models. However, some of the variables $(p_j, j = 1(1)n)$ in the objective functions in systems (1) and (2) are not in the constraints hence the models cannot be solved by simplex method or its variants. This is possibly why they were never solved in Rao (1990) and Nirmala and Jeeva (2012).

The mathematical formulation of our proposed model begins from equation (1) in Rao's model now restated as follows:

(a) The recruitment cost in period j is given by the concave function $z = k_i \delta p_j + v_j p_j$

$$\delta p_j = \begin{cases} 1 & if \quad p_j > 0 \\ 0 & if \quad p_j = 0 \end{cases} \qquad (3)$$

k_j is fixed recruitment cost in period j

v_j is the variable cost of recruitment per employee in period j

p_j is the number of staff recruited in period j

(b) The overstaffing cost is $l_j y_j$

l_j is cost of overstaffing per recruited staff in period

y_j is the number of staff recruited in an earlier period for the requirement of period j.

The total cost of recruitment for the n-period planning interval is:

$$Minimize\ \ z = \sum_{j=1}^{n}\left(k_j \delta p_j + v_j p_j + l_j y_j\right) \quad (4)$$

We take $y_0 = y_n = 0$ without loss of generality. The problem is to minimize this sum subject to the constraint that all requirements must be met on time, and since the variable cost of recruitment is constant we have that $\sum v_j\, p_j$ is a constant in equation (4). $\sum_{j=1}^{n} l_j p_j$ is also a constant because the point of its application depends on the earlier period at which recruitment took place and not period j. Furthermore $p_j y_j = 0, \forall j.$

Hence, the objective function in equation (1) becomes:

$$Minimize \;\; z = \sum_{j=1}^{n} \left[k\delta p_j + i_j y_j \right]$$

i.e. $$Minimize \; z = K + \sum_{j=1}^{n} i_j y_j \tag{5}$$

where $$K = \sum_{j=1}^{n} k\delta p_j \tag{6}$$

In equation (6), K is a fixed known cost for all periods.
Hence, equation (6) becomes:

$$Minimize \;\; z = \sum_{j=1}^{n} i_j y_j \tag{7}$$

Subject to the constraints:

$$\sum_{j=1}^{i} y_j \geq \sum_{j=1}^{i} R_j \, , \quad i = 1, \, 2, \cdots, n \tag{8}$$

The choice of the inequality '≥' is based on the assumption that overstaffing is allowed. Equation (8) implies that the total recruitment in period n should be greater than or equal to number of required staff. Thus the proposed DP model in LP form for determining the periodic recruitments (y_j) when $R_j \; \left(j = 1(1)n \right)$ are known is stated as follows:

$$\text{Minimize} \;\; z = \sum_{j=1}^{n} l_j y_j \tag{9}$$

s.t.

$$\sum_{j=1}^{i} y_j \; \geq \; \sum_{j=1}^{i} R_j \;\; , \;\; i = 1, \, 2, \cdots, n \tag{10}$$

$$y_j \geq 0 \;\; , \quad j = 1, \, 2, \cdots, n \tag{11}$$

Equation (9) is the objective function which is the total recruitment cost while equation (10) is the set of linear constraints with equation (11) as set of non-negativity constraints.

It can be seen in the proposed DP model in equations (9)-(11) that all the objective function variables are in the constraints of the problem, hence the problem is solvable, provided a feasible region exists.

The DP model in equations (9)–(11) is further transformed to the system (12) as primal DP model which also makes use of Table1

Primal DP Model

$$\left.\begin{array}{ll} Min\ z = l_1y_1 + l_2y_2 + l_3y_3 + \cdots + l_ny_n & \\ s.t. & \\ y_1 & \geq R_1 \\ y_1 + y_2 & \geq R_1 + R_2 \\ y_1 + y_2 + y_3 & \geq R_1 + R_2 + R_3 \\ y_1 + y_2 + y_3 + y_4 & \geq R_1 + R_2 + R_3 + R_4 \\ \cdots\cdots\cdots\cdots\cdots\cdots & \cdots\cdots\cdots\cdots\cdots\cdots \\ y_1 + y_2 + y_3 + \cdots + y_n & \geq R_1 + R_2 + R_3 + \cdots + R_n \\ y_1,\ y_2, \cdots, y_n \geq 0 & \end{array}\right\} \quad (12)$$

In quest for a DP model solution, we decide to formulate the dual of the DP model in system (12). The corresponding dual of the DP model in (12) is given in system (13).

Dual DP Model

$$\left.\begin{array}{l} Max\ \ w = R_1\sum_{i=1}^{n} d_i + R_2\sum_{i=2}^{n} d_i + R_3\sum_{i=3}^{n} d_i + \cdots + R_n\sum_{i=n}^{n} d_i \\ s.t. \\ \quad d_1 + d_2 + d_3 + \cdots + d_n \leq l_1 \\ \qquad d_2 + d_3 + \cdots + d_n \leq l_2 \\ \qquad\quad d_3 + \cdots + d_n \leq l_3 \\ \qquad\qquad \cdots\cdots\cdots \\ \qquad\qquad\qquad d_n \leq l_n \\ \quad d_i \geq 0, \quad i = 1(1)n \end{array}\right\} \quad (13)$$

where the $d_i's$ are the dual variables.

The system (13) is transformed to system (14) as follows:

$$\left.\begin{array}{l} Max\ \ w = R_1\sum_{i=1}^{n} d_i + R_2\sum_{i=2}^{n} d_i + R_3\sum_{i=3}^{n} d_i + \cdots + R_n\sum_{i=n}^{n} d_i \\ s.t. \\ \quad \sum_{i=k}^{n} d_i \leq l_k\ , \quad k = 1(1)n \\ \quad d_i \geq 0\ , \quad i = 1(1)n \end{array}\right\} \quad (14)$$

By deleting the first constraint/period (i.e starting from period 2) we obtain a primal sub-problem of (12) with a corresponding dual sub problem also obtained by deleting the first column in system (13). Continuing this way, we have n sub problem for n-periods manpower horizon. By backward recursive approach of DP, we start to determine by enumeration the dual suboptimal solution of the last nth period sub-problem and continue to the suboptimal solution of the first period

Let $D_k = \sum_{i=k}^{n} d_i$, $k = 1(1)n$

This ensures the non-negativity of the D_k since $d_i \geq 0,\ i = 1(1)n$. However, non-negativity of D_k does not imply that $d_i \geq 0,\ \forall\ i$, hence we impose additional constraints in equation (15).

$$D_k \geq D_{k+1},\ k = 1(1)n-1 \qquad (15)$$

$$\left.\begin{aligned} & Max\ \ w = \sum_{k=1}^{n} R_k D_k \\ & s.t. \\ & \quad D_k \leq l_k, \quad k = 1(1)n \\ & \quad D_k \geq D_{k+1},\ k = 1(1)n-1 \\ & \quad D_k \geq 0, \quad k = 1(1)n \end{aligned}\right\} \qquad (16)$$

which is the dual DP problem of the original primal DP problem.

In order to solve any of the dual sub-problems starting from the nth sub-problem, we rewrite the dual variables as follows:

The dual DP problem in system (16) is broken into n separate sub-problems and we start from the nth dual sub-problem using backward recursive approach.

The last sub-problem is given as:

$$\begin{aligned} & Max\ \ w = R_n D_n \\ & s.t. \end{aligned}$$

$$\left.\begin{aligned} D_n &\leq l_n \\ D_n &\geq 0 \end{aligned}\right\}$$ This gives the solution set $0 \leq D_n \leq l_n$

Note $D_n \geq D_{n+1}$ is the same as $D_n \geq 0$ because $D_{n+1} = 0$ as period $(n+1)$ does not exist.

The dual DP problem in (15) is now updated as follows:

Since we are maximizing $w, and\ R_k$ are known in Table 1,

$D_n = l_n$ or simply $D_n = \max(l_n\ ,\ 0) = l_n$

Similarly, the $(n-1)^{th}$ dual sub-problem is:

$$\left.\begin{aligned} & Max\ \ w = R_{n-1} D_{n-1} \\ & s.t. \\ & \quad D_{n-1} \leq l_{n-1} \\ & \quad D_{n-1} \geq D_n = l_n \\ & \quad D_{n-1} \geq 0 \end{aligned}\right\} \qquad (17)$$

The constraints in system (17) can produce solution set if $D_n = l_n \le l_{n-1}$ and $l_n \le D_{n-1} \le l_{n-1}$ i.e. $D_{n-1} = l_{n-1}$. In general if

$$l_k \le l_{k-1} \; , \; \left(k = 2(1)n\right) \qquad (18)$$

$$\text{then } D_k = l_k \; \left(k = 1(1)n\right) \qquad (19)$$

Substituting for D_k in the dual objective function, we have:

$$w = l_1 R_1 + l_2 R_2 + l_3 R_3 + \cdots + l_n R_n$$

$$= l_1 y_1 + l_2 y_2 + l_3 y_3 + \cdots + l_n y_n$$ of primal by Duality Theorem.

When the condition in (19) is satisfied, the solution is automatically $y_j = R_j$ (and R_j are given in Table 1). When the condition in equation (19) is not satisfied, it is advisable to solve the primal DP problem using a computer program for large size problems. The DP problem in system (18) is solvable if $l_n \le l_{n-1}$ and in general $l_k \le l_{k-1}$, $k = 2(1)n$. This means that the proposed model when used produces quick and accurate solution if condition in (19) is satisfied or using a computer program.

Numerical Illustration

Given the data in Table 2, determine how recruitment should be carried out throughout the planning period of the organization in order to minimize total recruitment cost.

Table 2: Hypothetical data for recruitment and overstaffing costs

Year N	**No. of Staff required** R	**Fixed Recruitment Cost k (₦)**	**Overstaffing cost i (₦)**
1	74	718	13
2	35	707	11
3	47	688	14
4	62	716	15
5	20	698	14
6	90	741	16
7	51	685	13
8	30	706	10
9	43	679	11
10	35	714	15

The DP model is as stated in system (9)-(11)

$$\left.\begin{aligned} &\text{Minimize} \quad z = \sum_{j=1}^{n} l_j y_j \\ &s.t. \\ &\qquad \sum_{j=1}^{i} y_j \geq \sum_{j=1}^{i} R_j \quad , \; i = 1, 2, \cdots, n \\ &\qquad y_j \geq 0 \quad , \quad j = 1, 2, \cdots, n \end{aligned}\right\} \qquad (9) \quad \text{-}(11)$$

Consequently, the linear programming problem is now formulated as a dynamic programming problem:

$$Minimize\, z = 13y_1 + 11y_2 + 14y_3 + 15y_4 + 14y_5 + 16y_6 + 13y_7 + 10y_8 + 11y_9 + 15y_{10}$$

$s.t.$

$$y_1 \geq 74$$
$$y_1 + y_2 \geq 109$$
$$y_1 + y_2 + y_3 \geq 156$$
$$y_1 + y_2 + y_3 + y_4 \geq 218$$
$$y_1 + y_2 + y_3 + y_4 + y_5 \geq 238$$
$$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 \geq 328$$
$$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 \geq 379$$
$$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 \geq 409$$
$$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9 \geq 452$$
$$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9 + y_{10} \geq 487$$
$$y_j \geq 0, \quad j = 1(1)10$$

Since the unit overstaffing costs l_k do not satisfy the condition

$$l_k \leq l_{k-1}, \quad k = 2(1)n \qquad (18)$$

Which is a limitation stated in section 3, the proposed model cannot be solved by backward recursive approach of DP technique. Consequently, we use the Program Full Simplex.

Note that in the solution process, the decision variables are rewritten as follows: $y_1 = x_1,\; y_2 = x_2, \cdots, y_{10} = x_{10}$

We present in Appendix A the Program Full-Simplex which is in PASCAL. The reason for using the computer program to solve the practical problem is that, apart from its high speed and accuracy (for a sparse LP), the DP problem has up to ten linear constraints and thirty variables in each of its tableau which makes it too complicated to solve manually.

Solution Process

The Program Full-Simplex is presented in appendix A

Table 3: Initial Tableau

Output

After compiling and running the program, the optimal solution is obtained at the 17th iteration in Table 4.

```text
Full simplex method
 phase i
  iteration  0
  base var.   Value   x1   x2   x3   x4   x5   x6   x7   x8   x9   x10   x11   x12   x13   x14   x15   x16   x17   x18   x19
x20   x21   x22   x23   x24   x25   x26   x27   x28   x29   x30
     x21      74.00  1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  -1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00
0.00  0.00  1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00
     x22      109.00  1.00  1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  -1.00  0.00  0.00  0.00  0.00  0.00  0.00
0.00  0.00  0.00  1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00
     x23      156.00  1.00  1.00  1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  -1.00  0.00  0.00  0.00  0.00  0.00
0.00  0.00  0.00  0.00  1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00
     x24      218.00  1.00  1.00  1.00  1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  -1.00  0.00  0.00  0.00  0.00
0.00  0.00  0.00  0.00  0.00  1.00  0.00  0.00  0.00  0.00  0.00  0.00
     x25      238.00  1.00  1.00  1.00  1.00  1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  -1.00  0.00  0.00  0.00
0.00  0.00  0.00  0.00  0.00  0.00  1.00  0.00  0.00  0.00  0.00  0.00
     x26      328.00  1.00  1.00  1.00  1.00  1.00  1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  -1.00  0.00  0.00
0.00  0.00  0.00  0.00  0.00  0.00  0.00  1.00  0.00  0.00  0.00  0.00
     x27      379.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  -1.00  0.00
0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  1.00  0.00  0.00  0.00
     x28      409.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  -1.00
0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  1.00  0.00  0.00
     x29      452.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  -
1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  1.00  0.00
     x30      487.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00
0.00  -1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  1.00
     z      0.00   13.00  11.00  14.00  15.00  14.00  16.00  13.00  10.00  11.00  15.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00
0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00
     -w     -2850.00 -10.00  -9.00  -8.00  -7.00  -6.00  -5.00  -4.00  -3.00  -2.00  -1.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00  1.00
1.00  1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00
```

Table 4: Optimal Tableau (17th Iteration)

```text
 I
  BASE VAR.   VALUE   X1   X2   X3   X4   X5   X6   X7   X8   X9   X10   X11   X12   X13   X14   X15   X16   X17
X18   X19   X20
     X1      74.00  1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  -1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00
0.00
     X2      305.00  0.00  1.00  1.00  1.00  1.00  1.00  1.00  0.00  0.00  0.00  1.00  0.00  0.00  0.00  0.00  0.00  -1.00  0.00
0.00  0.00
     X12      270.00  0.00  0.00  1.00  1.00  1.00  1.00  1.00  0.00  0.00  0.00  0.00  1.00  0.00  0.00  0.00  0.00  -1.00  0.00
0.00  0.00
     X13      223.00  0.00  0.00  0.00  1.00  1.00  1.00  1.00  0.00  0.00  0.00  0.00  0.00  1.00  0.00  0.00  0.00  -1.00  0.00
0.00  0.00
     X14      161.00  0.00  0.00  0.00  0.00  1.00  1.00  1.00  0.00  0.00  0.00  0.00  0.00  0.00  1.00  0.00  0.00  -1.00  0.00
0.00  0.00
     X15      141.00  0.00  0.00  0.00  0.00  0.00  1.00  1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  1.00  0.00  -1.00  0.00
0.00  0.00
     X16      51.00  0.00  0.00  0.00  0.00  0.00  0.00  1.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  1.00  -1.00  0.00
0.00  0.00
     X8      108.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  1.00  1.00  1.00  0.00  0.00  0.00  0.00  0.00  0.00  1.00  0.00
0.00  -1.00
```

X18	78.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	-1.00
X19	35.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	-1.00
z	-5397.00	0.00	0.00	3.00	4.00	3.00	5.00	2.00	0.00	1.00	5.00	2.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	10.00

Discussion

From iteration 17 (optimal tableau), the optimal solution in terms of the original variables is:

$$\mathbf{y_1 = 74, y_2 = 305,}\ y_{12} = 270,\ y_{13} = 223,\ y_{14} =$$
$$\mathbf{y_8 = 108,}\ y_{18} = 78,\ y_{19} = 35\ \textit{and}\ \mathbf{z = ₦5,397}$$

The bolded values are the three decision variables (y_1, y_2 *and* y_8) and the objective function value. The total number of staff recruited in periods 1, 2, and 8 is $y_1 + y_2 + y_8 = \mathbf{487}$ and using the objective function we obtain $l_1 y_1 + l_2 y_2 + l_8 y_8$ = ₦5,397 which is equal to the objective function value in the optimal tableau.

The optimal solution to the proposed DP model in LP form for the given example reveals that out of the ten basic variables in the optimal tableau (iteration 17), seven of them which are surplus are non-decision variables while the remaining three variables are decision variables that contribute to the objective function value. It is interesting to note that the three decision variables constitute a total staff recruitment of 487. This is equal to the 487 recruited staff obtained from both Rao's model and our proposed DP model.

Furthermore, while Rao's DP model yielded ₦5,757 as minimum total recruitment cost, that of our proposed DP model in LP form for the same problem yielded ₦5,397. The difference of ₦360 is certainly the constant cost K which is part of the objective function in section 3, but for obvious reason should not be included in the LP objective function. That is, the constant K, which is the payable bulk fixed recruitment cost is the difference between the objective function.

Conclusion

We have been able to formulate a manpower planning problem based on only recruitment factor as a DP problem in LP form which has the advantage of quick and accurate solution over that of Rao's (1990) model due to its sparse features which makes it possible for computer implementation using Program Full-Simplex.

REFERENCES

Bontis, N., Dragonetti, K. and Roos, G. (1999), "The Knowledge Tool Box: A Review of the Tools Available to Measure and Manage Intangible Resources". European Management Journal, 17(4), 391-302.

Biruk, S., Jaskowski, P. and Maciaszczyk, M. (2022): Conceptual framework of simulation based Manpower Planing Method for construction Enterprises, available online at https://www.mdpi.com/Journal/sustainability

Cai, X., Minghni, L. and Li, Y. (2010), "Stochastic Manpower Planning by Dynamic Programming". 2nd Internal Conference on Engineering Optimization, Lisbon, Portugal.

Cole, G.A. (2005), "Personal and Human Resources Management,"Bookpower/ELST, U.K.

Ezugwu, V.O and Olugun, S. (2017): Markov chain: A predictive model of manpower planning *Journal of applied sciences, Environment, Management*, 21 (3), 557-565

Gregoriades, A. (2000), "Manpower Planning with System Dynamics and Business Objects". Available online at http://www.google.com.

Mehlmann, S.A. (1980), "An Approach to Optimal Recruitment and Transition Strategies for Manpower System Using Dynamic

Mutingi, M. and Mbohwa, C. (2012), "Fuzzy System Dynamics and Optimization with Application to Manpower Systems" Available at: www. Growing Science .Com

Nirmala, S. and Jeeva, M. (2010), "Dynamic Programming Approach to Manpower Recruitment and Promotion Policies for a two Grade System". African Journal of Mathematics and Computer Science Research vol. 3(12), 297-310.

Ogumeyo, S.A. (2010), "Optimum Workforce-size Model Using Dynamic Programming Approach". *Global Journal of Mathematical Sciences*,*9* (1), 11-16.

Parker, B. (1996), "Holonic Modelling: Human Resources Planning and Two faces of Janus", *Int. Journal of Manpower*, 17(8), 30-45.

Ranslam P. (2022). Understaffing and overstaffing-A delicate balance available online at factorialhr.com

Rahela, A.R. (2015). Analyzing Manpower Data of higher learning institutions; A Markov chain approach. *International journal of human resource studies,* 5(3), 38-47

Udom, A.U ; Ebedoro,(2021) U.G. on multinomial hidden Markov model for hierarchical manpower systems. Commun. Stat. theory methods, 50, 1370-138

Valier, K. (2023): A Three Stage Manpower Planning and Schedulind Model available online at *www.ressearchgate.net*

Verbeken, B. and Guerry, M. (2021): Discrete time hybrid semi- Markov Models in Manpower planning available online at *https://www.mdpi.com/Journal/mathematics*

Wagner, H.M. (2001), "Principle of Operations Research," New Delhi: Prentice Hall of India.

Wan-yin, D. and Shou, L. (2015), Application of Markov Chain Model in Human Resources Supply Forecasting Enterprises. *International Conference on Computational Science and Engineering.*

Yan, S.Y. and Chen C.k. (2008), "Short-term Shift Setting and Manpower Supplying Under Stochastic Demands for Air". Transportations, 35(3), 425-444.

Appendix A

Program FullSimplex

```
PROGRAM FullSimplex  (f3,f4);
USES CRT;
CONST
   nvar=12; m=6;      { No, of variables and constraints }                    (**)
   ncols=18;                   { Maximum no. of columns in tableau }          (**)
   fwt=7; dpt=2;    { Output format constants for tableau values }            (**)
   fwi=1;              { Ouput format constant for indices }                  (**)
   largevalue = 1.0E20;  smallvalue=1.0E-10;                                  (**)

TYPE      mrange = 1..m;   ncolsrange  =  1..ncols;
   matrix  =  ARRAY [mrange, ncolsrange] OF real;
   column = ARRAY [ mrange] OF real;
   baseindex = ARRAY [mrange] OF integer;
   row = ARRAY [ncolsrange] OF real;
  rowboolean = ARRAY [ncolsrange] OF boolean;
  phase = (PhaseI, phaseII);

VAR
a : matrix;       { Matrix A in standard form of problem, see (2.3)  }
b : column;       { Vector b in standard form of problem, see (2.3)  }
c : row;          { Coefficients of objective function, see (2.1)  }
d : row;          { Coefficients of artificial objective function   }
basic :  baseindex;             { Basic variables at each stage }
nonbasic  : rowboolean;     { Status indicators for variables }
w0, z0 : real;                     { Values of objective functions }
it  :  integer;         { Iteration counter }
solution, OK : boolean; { Iteration process terminators }
r, s :  integer;        { Row and column of pivot element }
GC, EC, LC : integer;        { No. of  ´>= ´,  ´= ´  and   ´<= ´  constraints  }
n1, n2, GCplusLC, GCplusEC:  integer;
printon  : boolean;   i : integer;   slack : row;
f3,f4:text;

PROCEDURE inputdata;
VAR     i, j,  k : integer;
BEGIN
read(f3,k); printon:=k>0;  read(f3,GC,EC,LC);
FOR  i:=1  TO  m  DO
BEGIN FOR j:=1 TO nvar DO read(f3,a[i,j]);  read(f3,b[i])  END;
FOR  j:=1  TO nvar DO read(f3,c[j])
END;  { inputdata }

PROCEDURE initialise;
VAR   i, j :  integer;
BEGIN it:=0; z0:=0.0; OK:=true; GCplusLC:=GC+LC; GCplusEC:=GC+EC;
```

```
    n1 := nvar + GCplusLC + GCplusEC; n2 := nvar + GCplusLC;
    FOR  j := nvar+1 TO n1 DO
    BEGIN FOR i:=1 TO m DO a[i, j] :=0.0; c[ j ]:=0.0 END
END; { initialise }

PROCEDURE  completetableau;
VAR  i, j : integer;  sum  : real;
BEGIN   FOR  i:=1  TO  GC  DO a[i,nvar+i]  := -1.0;
    FOR  i:=1 TO  LC  DO  a[GCplusEC+i,nvar+GC+i]  := 1.0;
    FOR  i:=1  TO GCplusEC  DO a[i,nvar+GCplusLC+i]  := 1.0;
    {  Compute initial base  }
    FOR j:=1 TO GCplusEC DO basic[j] := nvar + GCplusLC + j;
    FOR j:=1 TO LC DO basic[GCplusEC+j]  :=  nvar + GC  +  j;
    FOR j:=1 TO n1 DO nonbasic [j] := true;
    FOR i:=1 TO m DO nonbasic [basic[i]] := false;
{ Compute d-values and w0 }
FOR  j:=1 TO n2 DO
BEGIN  sum :=0.0;
    FOR  i:=1 TO GCplusEC  DO sum := sum  + a[i , j] ; d[j] :=-sum
END;  sum :=0.0;
FOR j:=n2+1  TO n1 DO d[j] := 0.0;
FOR i:=1 TO GCplusEC  DO  sum  := sum + b[i];   w0:=-sum
END; { completetableau }


PROCEDURE outputtableau (p:phase);
VAR  i, j, n : integer;
BEGIN  IF p=phaseI THEN n:=n1 ELSE n:=n2;
     writeln (f4); writeln(f4,'  ITERATION ',  it:2);
write (f4, '   BASE VAR.   ',  ' ':fwt-5,   'VALUE');
FOR j:=1  TO n  DO write(f4,' ':fwt-fwi-1,  'X', j:fwi);  writeln(f4);
FOR i:=1 TO m  DO
BEGIN
     write(f4,' ' :8-fwi,  'X', basic[i]:fwi,  ' ':7,  b[i]:fwt:dpt);
FOR  j:=1  TO  n DO  write (f4,a[i , j] : fwt:dpt);  writeln (f4)
END;
write(f4,' ':7,  'z',  ' ':7, z0:fwt:dpt);
FOR j:=1  TO n DO write(f4,c[j]:fwt:dpt);    writeln (f4);
IF p=PhaseI THEN
BEGIN   write (f4,' ':7,  '-w',   ' ':7, w0:fwt:dpt);
    FOR  j:=1 TO n DO write(f4,d[j] :fwt:dpt) ;   writeln (f4)
END
 END;   { outputtableau  }



PROCEDURE Simplex (p:phase);
VAR   n  :  integer;  unbounded : boolean;
```

```
PROCEDURE nextbasicvariable (VAR  r,s:integer; x:row) ;
VAR   i , j : integer;  min : real;
BEGIN  min:=largevalue;  { Find the variable,  s, }
   FOR  j:=1  TO  n DO  {  to enter the basis.   }
    IF nonbasic [j]  THEN  IF x[j]<min  THEN  BEGIN min:=x[j]; s:=j END;
    solution := x[s]  > -smallvalue;
    IF  NOT solution THEN
    BEGIN  unbounded :=true; i:=1;       { check that at least one value }
        WHILE  unbounded AND (i<=m)  DO  { in column s is positive.     }
        BEGIN  unbounded  := a [i,s] < smallvalue; i:=i+1     END;
        IF NOT unbounded THEN
        BEGIN  min:=largevalue;      { Find the variable,  basic[r],  }
          FOR  i:=1  TO  m  DO        { to leave the basis.                          }
           IF  a[i,s]  > smallvalue THEN
                        IF b[i]/a[i,s] < min THEN BEGIN  min:=b[i]/a[i,s]; r:=i  END;
          nonbasic[basic[r]]:=true; nonbasic[s]:=false; basic[r]:=s; writeln(f4);
            writeln(f4,'   PIVOT IS AT ROW  ', r:fwi,  ' COL ' , s:fwi)
         END
       END
 END;    {  nextbasicvariable  }

PROCEDURE  transformtableau (r, s:integer; VAR x:row; VAR x0:real);
{  Construct the new canonical form, implementing (2.15) to (2.20)  }
VAR      i , j : integer;  pivot, savec, savex : real;  savecol : column;
BEGIN
    FOR i:=1  TO m DO savecol[i] :=a[i , s] ;  savex:=x[s];   pivot:=a[r, s];
    b[r]:=b[r]/pivot;            {(2.15) }
    FOR j:=1 TO  n  DO a[r,j] :=a[r, j]/pivot;  { (2.16)}
    FOR i:=1 TO m DO
      IF i<>r THEN
  BEGIN b[i]:= b[i] - savecol[i]*b[r]; {(2.17)}
    FOR j:=1 TO n DO a[i,j] := a[i,j] - savecol[i]*a[r,j]  {(2.18)}
   END;
  FOR j:=1 TO n DO x[j]  := x[j] - savex*a[r, j] ;   {(2.19)}
  x0  := x0 - savex*b[r];  {(2.20)}  it := it+1;
  IF p=PhaseI THEN
  BEGIN savec:=c[s];  FOR j:=1  TO n DO c[j] := c[j] -savec*a[r,j];
   z0 := z0 - savec*b[r]
  END
END;     {transformtabeau }

BEGIN { Simplex }
    solution:=false; unbounded:=false;
    IF  p=PhaseI THEN n:=n1 ELSE n:=n2; { Determine current tableau size }
    REPEAT
        IF printon THEN outputtableau(p);
       CASE p OF
       PhaseI    :  nextbasicvariable(r, s, d);
             PhaseII   :   nextbasicvariable(r, s, c)
```

```
        END;
        IF NOT (solution OR unbounded) THEN
          CASE p OF
                      PhaseI   : transformtableau(r, s, d, w0);
                      PhaseII  : transformtableau(r, s, c, z0)
                  END
     UNTIL solution OR unbounded;
     IF  unbounded THEN writeln(f4,'   UNBOUNDED')
END;  { Simplex }

BEGIN { Main Program }
assign(f3,'C:\Dev-Pas\4indata.txt'); reset(f3);
assign(f4,'C:\Dev-Pas\4outdata.txt'); rewrite(f4);
    writeln(f4); writeln(f4,'    FULL SIMPLEX METHOD'); writeln(f4);
    inputdata; initialise;  completetableau;
    IF GCplusEC=0 THEN writeln(f4,'    THERE IS NO PHASE I')
    ELSE    { Perform Phase I }
    BEGIN   writeln(f4,'    PHASE I');
       Simplex(PhaseI);   writeln(f4);
       IF (abs(w0) >smallvalue) OR (NOT solution) THEN
       BEGIN  OK:=false;   writeln(f4,'   PHASE I NOT COMLETED');
                    writeln(f4,'  SUM OF ARTIFICIALS ', w0:fwt,dpt)
       END
       ELSE
       BEGIN  writeln(f4);   writeln(f4,'   PHASE I SUCCESSFUL'); writeln (f4);
            writeln(f4,'   REDUCED TABLEAU FOR PHASE II')
      END
   END;
   IF OK THEN   { perform Phase II }
   BEGIN Simplex(PhaseII); writeln(f4);
      IF NOT solution THEN writeln(f4,'   PHASE  II NOT COMPLETED')
      ELSE

    BEGIN   { Output final details }
              writeln(f4); writeln(f4,'   FINAL SOLUTION'); writeln;
              writeln(f4,'    MINIMUM  Of  Z = ', -z0:fwt:dpt); writeln;
      writeln(f4,'    CONSTRAINT BASIS   VALUE    STATE    SLACK');
      FOR i:=1 TO m DO slack [basic[i]] := b[i];
      FOR i:=1 TO m DO { For each constraint }
             BEGIN   write(f4,i:10, basic[i] :10, ' ':12-fwt, b[i]:fwt:dpt,  ' ':5);
                 IF (i <=GC) OR (i>GCplusEC) THEN
                   IF nonbasic[nvar+i] THEN writeln('BINDING', 0.0:10:dpt)
          ELSE  writeln(f4,'SLACK',' ':12-fwt, slack[nvar+i] :fwt:dpt)
                ELSE  writeln(f4,'EQUATION    NONE')
     END;
    END;
   END;
END.  { FullSimplex }
```