# SENSITIVITY ANALYSIS IN A MANPOWER PLANNING MODEL

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**ABSTRACT**

Sensitivity analysis deals with the investigation into various changes in the optimum solution of a model due to changes in the original data. This research is an extension of the manpower planning models which use linear and dynamic programming techniques to determine optimum recruitment and wastage cost without incorporating sensitivity analysis in their formulations. This work aims at correcting this shortcoming by incorporating sensitivity analysis in the model using linear and dynamic programming techniques which investigate various changes in the objective function coefficients and changes in the right hand side values of the constraints. A numerical example using computer program simplex has been presented to demonstrate the effects of these changes. It is observed that the number of staff recruited and retrenched or retired are equal to the maximum number of staff anticipated (H) in the manpower system. It is also observed that the objective function value is highest when H is increased by two units and the initial number of staff (h) is increased by one unit.

**Keywords:** Recruitment, Wastage, Linear Programming, Dynamic Programming, Sensitivity Analysis.

## Introduction

Sensitivity analysis has been defined as the "what if" analysis where we investigate what happens to the present optimal solution if certain changes occur in the constant parameters of the original problem without having to solve the new problem afresh. Investigations that deal with the changes in the optimum solution due to changes in the original data can represent either real changes that can be made in the operation of the physical system which the model represents or fictitious changes which are made to investigate the effects of uncertainty in the basic data Taha (2002).

In mathematical modeling, we do take cognizance of significant controllable and uncontrollable variables as well as parameters of the model, Ravi and Wendel (1985), Gabriel and Teodorescu (2022) and Kenton (2023). The parameters are input variables which help to specify the relationship between other types of variables and for a given simulation the parameters have a constant value. In sensitivity analysis, as remarked in Lucey (1996), the model input data is used to find not only an optimal solution, but also to determine what happens to the optimal solution when certain changes are made in the system and the effects of these changes without having to solve a new problem or a series of new problems.

Despite the advantages of good planning that accrue in having the knowledge of the effects of changes in values of the model input on the output variables, factors such as time , cost and risk often discourage firms from carrying out sensitivity analysis of system models as stated in Greenwald and Stiglitz (1990). Based on this risk factor, the differences in observed risk propensity and their impact on firm performance are explored in Smith and Nau (1999). A decision theoretical model which (a) measures a firm's risk propensity in the form of an "implied" utility function (b) investigates changes in corporate risk propensity with respect to changes in firm size and (c) examines the relationships between firm's risk propensities and alternative dimensions of economic performance is developed in Walls and Dyer (1996). The risk of a company's income stream for a given year by the variance in security analysts' forecasts of that income is discussed in Bromiley (1991).

Models containing sensitivity analysis in literature in recent times are: Sensitivity analysis for inverse probability weighing estimators via the percentile bootstrap, (Zhao et al 2019), Bias formula for sensitivity analysis of unmeasured confounding general outcomes, treatment, and confounders' epidemiology (Vander and Arah, 2011), A flexible sensitivity analysis for observational studies without observable implications (Frank et al, 2019), A selection bias approach to sensitivity analysis for causal effects, (Blackwell, 2014), Sensitivity analysis in observational research: introducing the E-value, (Vander, Ding, 2017).

Sensitivity analysis can be carried out in LP model and the changes in the LP problem that are usually investigated include:(1) changes in the Right Hand Side (RHS) values, (ii) changes in the coefficients of the objective function and (iii) changes in the coefficients of the matrix.

This research is an extension of the manpower planning models in Rao (1990), Ogumeyo and Ekoko (2015) and Ogumeyo and Okogun (2023) in which optimum recruitment and wastage costs were considered without reference to sensitivity analysis. This work aims at correcting this shortcoming by incorporating sensitivity analysis in these manpower planning models which serves as a tool to investigate the effects of the changes in the Right Hand Side (RHS) values, and (ii) changes in the coefficients of the objective function.

**Model Parameters and Methodology**

Model description: Let $y_j(t+\delta)$ be the number of staff recruited at time $(t+\delta)$ of period $j$ where $\delta$ is the very small time difference between recruitment and assumption of duty so that the recruited staff arrive at time $(t+\delta)$ for work. Let $x_j(t+\delta)$ *and* $c_j(t+\delta)$ be the number of staff on wastage and the average accrued revenue to the organization from each wastage staff in period $j$ by virtue of their exit from the organization. Let $c'_j(t+\delta)$ be the average salary per recruited staff at time $(t+\delta)$ of period $j$ when the recruitment was done. As $\delta \to 0$, the above notations become $x_j(t), y_j(t), c_j(t)$ *and* $c'_j(t)$ or simply $x_j$ , $y_j, c_j$ *and* $c'_j$. Given $h, H, c_j(t)$ and $c'_j(t)$ of a manpower planning problem, it is required to determine the optimal quantities $x_j$ *and* $y_j$ so that the accruable net revenue is a maximum. As we are dealing here with a dynamic situation, we divide the time span of interest into time intervals, which we shall assume to be sufficiently short so that we can consider $x_j(t), y_j(t), c_j(t)$ *and* $c'_j(t)$ to be constants during the time intervals but discontinuous from one time interval to the next.

**Notations**

$x_j$ = number of staff that are on wastage in period $j$.

$y_j$ = number of staff that are recruited in period $j$.

$c_j$ = average accrued revenue to the organization from each wastage staff in period $j$ by virtue of their exit from the system.

$c'_j =$ average salary per recruited staff in period $j$.

$h =$ initial number of staff on ground in the organization at the beginning of the time horizon.

$H =$ maximum number of staff anticipated at the end of the time horizon under consideration.

**Assumptions of the Model:** It is assumed that

$h =$ minimum number of staff required by organization to takeoff is known.

$H =$ maximum number of staff anticipated in the manpower system is known.

**Model Formulation**

The problem of the manpower planning is to maximize the periodic additional revenue accruable to the organization from the wastage staff wage bill less the periodic salary of recruited staff i.e. $\sum_{j=1}^{n}\left(c_j x_j - c'_j y_j\right)$.

The objective function can be written as:

$$Maximize\ z = \sum_{j=1}^{n}\left(c_j x_j - c'_j y_j\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

There are two sets of staffing constraints and two sets of non-negativity constraints in this manpower planning problem.

(i) The overstaffing constraints:

The constraints of overstaffing state that the total number of overstaffing staff of the first $i$ periods should not exceed the available vacancies $(H-h)$ in the establishment, i.e.

$$\sum_{j=1}^{i}\left(y_j - x_j\right) = -\sum_{j=1}^{i} x_j + \sum_{j=1}^{i} y_j \le H - h, \quad i = 1(1)n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Where $(y_j - x_j) > 0$ is the number of staff by which the organization is overstaffed in period $j$

The LHS of equation (2) can also be called the net increase in manpower in the first $i$ periods.

(ii) The understaffing constraints:

The constraints of understaffing represent the number of staff by which the organization is understaffed for the first $(i-1)$ periods plus wastages at period $i$ and this should not exceed $h$ the number of staff originally in the organization. If it does, it means the organization has only material resources which is not the case in practical situation as existence of an organization is based on the contribution of human and material resources. Mathematically this is expressed as:

$$\sum_{j=1}^{i-1}(x_j - y_j) + x_i = \sum_{j=1}^{i} x_j - \sum_{j=1}^{i-1} y_j \leq h, \ \ i = 1(1)n \qquad (3)$$

Where $(x_j - y_j) > 0$ is the number of staff by which the organization is understaffed in period $j$

The L.H.S of equations (3) can also be called the net increase in manpower subtracted from wastage staff in the first $(i-1)$ periods plus the wastage manpower in period $i$.

Note that the second summation in equation (3) does not exist for $i = 1$

(iii) Non-negativity constraints:

The non-negativity constraints are

$$x_j,\ y_j \geq 0, \ \ j = 1(1)n \qquad (4)$$

Equation (1) stated above constitutes the total manpower planning cost from all the $n$ periods while equations (1)-(4) constitute a DP problem which is stated thus:

**Primal LP Problem**

$$\left.\begin{aligned} &Maximize \quad z = \sum_{j=1}^{n}\left(c_j x_j - c'_j y_j\right) \\ &s.t. \\ &\quad -\sum_{j=1}^{i} x_j + \sum_{j=1}^{i} y_j \leq H - h, \quad i = 1(1)n \\ &and \quad \sum_{j=1}^{i} x_j - \sum_{j=1}^{i-1} y_j \leq h, \quad i = 1(1)n \\ &\qquad x_j\,, \ \ y_j \geq 0 \ \ , \quad j = 1(1)n \end{aligned}\right\} \qquad (5)$$

The system (5) is the DP model of the manpower planning problem which makes use of both recruitment and wastage factors. The DP model in system (5) has 2n linear constraints, 2n nonnegativity constraints in 2n variables. Further simplification of (5) yields the system in (6).

$$
\left.
\begin{array}{llll}
Max\ \ z = c_1x_1 + c_2x_2 + \ldots + c_nx_n - c_1'y_1 - c_2'y_2 - \ldots - c_n'y_n & & & \\
s.t. & & & \\
\quad -x_1 & +y_1 & \le & H-h \\
\quad -x_1 - x_2 & +y_1 + y_2 & \le & H-h \\
\quad -x_1 - x_2 - x_3 & +y_1 + y_2 + y_3 & \le & H-h \\
\quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & & & \\
\quad -x_1 - x_2 - x_3 - \ldots - x_n & +y_1 + y_2 + y_3 + \ldots + y_n & \le & H-h \\
\quad x_1 & & \le & h \\
\quad x_1 + x_2 & -y_1 & \le & h \\
\quad x_1 + x_2 + x_3 & -y_1 - y_2 & \le & h \\
\quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & & & \\
\quad x_1 + x_2 + x_3 + \ldots + x_n & -y_1 - y_2 - y_3 - \ldots - y_{n-1} & \le & h \\
\quad x_j,\ y_j \ge 0,\ \ j = 1(1)n & & &
\end{array}
\right\} \qquad (6)
$$

The matrix skeleton of the system (6) is shown in Fig.1

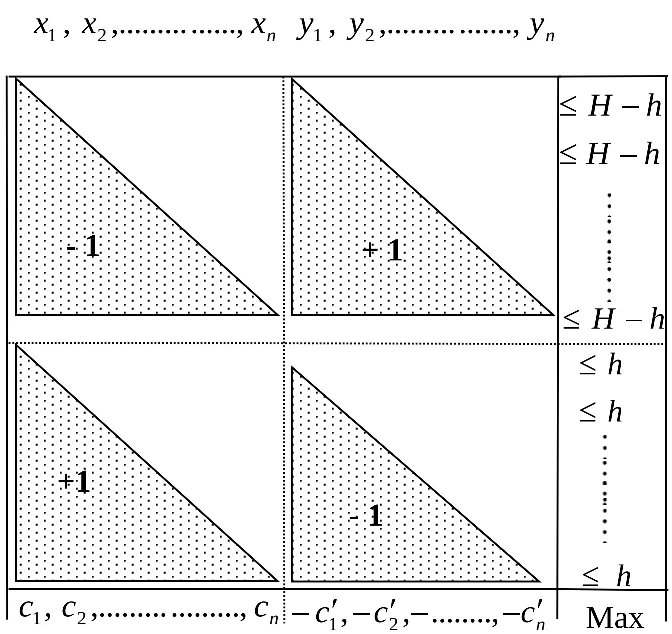


**Fig. 1:** Matrix skeleton of the primal LP model coefficient arrays.

The matrix array consists of triangular blocks, which are typical of dynamic situations. The coefficients within the blocks are either $+1\ or\ -1$ depending on the block. The triangular block in the lower right-hand corner is smaller by one row and one column than the other three.

Let $d_1, d_2, \cdots, d_n$ be the first n dual variables for the first n constraints in system (6) and $e_1, e_2, \cdots, e_n$ be the last n dual variables for dual DP model of the manpower planning problem:

**Dual DP Problem**

$$Minimize\ w = (H-h)\sum_{i=1}^{n} d_i + h\sum_{i=1}^{n} e_i \qquad (7)$$

s.t.

$$-\sum_{i=k}^{n} d_i + \sum_{i=k}^{n} e_i \geq c_k, \quad k = 1(1)n \qquad (8)$$

$$\sum_{i=k}^{n} d_i - \sum_{i=k+1}^{n} e_i \geq -c'_k, \quad k = 1(1)n \qquad (9)$$

$$d_i, e_i \geq 0, \quad i = 1(1)n \qquad (10)$$

It is understood that the second summation in equation (9) does not exist if $k = n$. The corresponding matrix skeleton of the dual DP problem in equations (7)–(10) is shown in Fig. 2

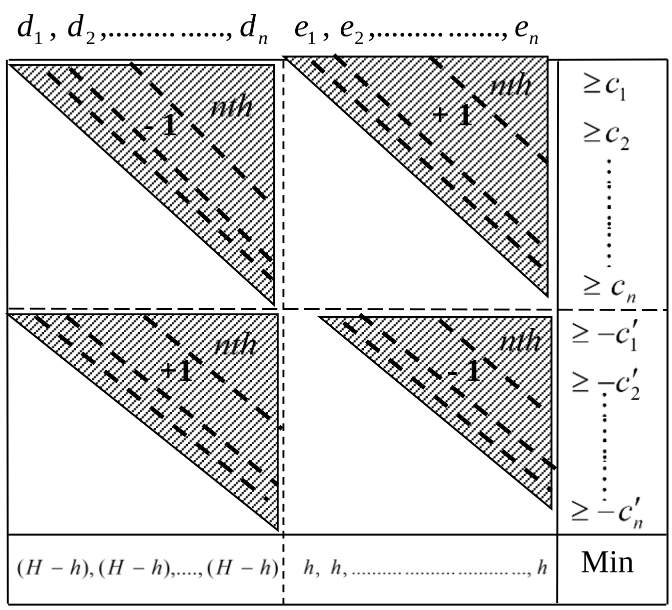


**Fig. 2:** Matrix skeleton of the dual DP model coefficient arrays.

We define new variables $D_k$ *and* $E_k$ as follows:

$$D_k = \sum_{i=k}^{n} d_i, \quad k = 1(1)n \qquad (11)$$

$$E_k = \sum_{i=k}^{n} e_i, \quad k = 1(1)n \qquad (12)$$

Since by the dual DP problem, $d_i$ *and* $e_i$ are nonnegative, $D_k$ *and* $E_k$ must be nonnegative. However, non-negativity of $D_k\ and\ E_k$ does not imply that $d_i \geq 0\ and\ e_i \geq 0,\ \forall\ i$. In view of the definition of $D_k\ and\ E_k$, we see that non-negativity of $d_i\ and\ e_i$ will be ensured if we augment the dual LP problem, expressed in terms of $D_k\ and\ E_k$ by the constraints:

$$D_k \geq D_{k+1}\ ,\quad k = 1(1)\ n-1 \qquad (13)$$

$$E_k \geq E_{k+1}\ ,\quad k = 1(1)\ n-1 \qquad (14)$$

This is the dual DP problem starting with period 1 while $D_1\ and\ E_1$ are the smallest values in their solution set.

**3.0 Sensitivity Analysis of the Manpower Planning Model**

There is need to examine the effects of different changes in the number of employees initially on ground and at the end of the time horizon denoted by $h\ and\ H$ respectively.

**(1) Effect of keeping H unchanged and increasing only h:**

**Theorem 1**

If H is kept unchanged and $h$ is increased to H, then the objective function value (z) of the primal LP problem is increased to

$$z' = z + (H-h)(E_1 - D_1) \qquad (15)$$

**Proof**

When $h$ is increased by 1 unit, each of the upper half constraints of the primal LP model in system (6) is reduced by 1 unit while each constraint in the lower half is increased by 1 unit. The resultant effect is the increase by $(\sum_{i-1}^{n} e_i - \sum_{i=1}^{n} d_i)$ of the primal objective function value (z). When $h$ is increased to H, i.e. increased by $(H-h)$, the new primal objective function value is $z' = z + (H-h)(E_1 - D_1)$, where $E_1 = \sum_{i=1}^{n} e_i\ and\ D_1 = \sum_{i=1}^{n} d_i$ as earlier defined. This completes the required proof. This theorem will be numerically illustrated in section four.

**Note:** The primal objective function is $z = \sum_{j=1}^{n}(c_j x_j - c'_j y_j)$. When $(E_1 - D_1) > 0$, the net accruable revenue to the organization from human resources is increased per unit increase in $h$ which is an advantage. The financial increment can be enhanced up to $(H-h)(E_1 - D_1)$ when $h$ is increased very closed to $H$. It will be economically disadvantageous to the organization to increase $h$ (i.e. to start with higher periods with $h' = h + p,\ 0 < p \leq (H-h)\ if\ (E_1 - D_1) < 0$.

**(2) Effect of reducing H by two units and increasing h by one unit.**

In this case, both the upper half and lower half constraints in system (6) will be increased by one unit. This leads to a higher new primal objective function value

$$z' = z + (H-h)(E_1 + D_1) \quad (16)$$

This is so because there will be more output through increased upper boundary limits of all manpower constraints making it possible for the organization to attain full capacity production earlier than anticipated.

**(3)** Similarly, the following effects are considered.

**(a) Effect of increasing H and keeping h unchanged**

$$z' = z + \sum_{i=1}^{n} d_i = z + D_1 \quad (17)$$

where $D_1$ is the financial increase per unit increase in $H$ when h is unchanged.

**(b) Effect of increasing both h and H**

$$z' = z + \sum_{i=1}^{n} e_i = z + E_1 \quad (18)$$

Where $E_1$ is the financial increase per unit increase in both h and H.

**(c) Effect of reduction in both h and H**

$$z' = z - \sum_{i=1}^{n} e_i = z - E_1 \quad (19)$$

where $E_1$ is the financial decrease per unit decrease in both h and H. This should be completely avoided because the objective function value (z) which we seek to maximize is to be further reduced by $E_1$.

**Numerical Illustration**

Links between personnel and vacancy flows in a graded personnel system, focusing on outside hiring within a university community is discussed in Ogumeyo and Ekoko (2015). The need for models that should estimate projected manpower for between ten and twenty years planning horizon is emphasized in Feuer and Schinnar (2017) and Frank and Feller (2019). Based on these critical remarks we have obtained the following data from one of the tertiary institution in Nigeria for both junior and senior staff.

**Table 1:** Average monthly salary for **junior staff** on wastage and recruitment

| Year | 2011 (1) | 2012 (2) | 2013 (3) | 2014 (4) | 2015 (5) | 2016 (6) | 2017 (7) | 2018 (8) | 2019 (9) | 2020 (10) | 2021 (11) | 2022 (12) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $c_j$ | 33286 | 32045 | 35770 | 35918 | 36637 | 37552 | 38437 | 39126 | 33065 | 32281 | 38084 | 40124 |
| $c'_j$ | 30148 | 32281 | 33665 | 34305 | 37545 | 34305 | 37894 | 36157 | 32981 | 30467 | 37688 | 36645 |

For Table 1, h=162 and H=393

Table 2**:** Average monthly salary for senior staff on wastage and recruitment

| Year | 2011 (1) | 2012 (2) | 2013 (3) | 2014 (4) | 2015 (5) | 2016 (6) | 2017 (7) | 2018 (8) | 2019 (9) | 2020 (10) | 2021 (11) | 2022 (12) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $c_j$ | 127104 | 131223 | 135211 | 140421 | 142995 | 159213 | 162372 | 179084 | 180512 | 182750 | 184152 | 187289 |
| $c'_j$ | 74372 | 76911 | 80625 | 83179 | 88370 | 91372 | 94246 | 96960 | 99124 | 102629 | 113893 | 118413 |

For Table 2, h=230 and H=600

We considered average monthly salary for a period of up to 12 years so that our results can give good estimates of staff wastage $x_j$ and recruitment $y_j$. Based on the present salary trend, we want to determine the optimal annual number of staff on wastage and recruitment that will maximize total accruable revenue to the institution in the next 12 years (i.e. by the year 2034) when the senior staff strength is planned to be 600.

**Solution by Linear Programming (LP) Approach**

The primal LP model based on wastage and recruitment factors (for senior staff) is given below with the optimal table after 24 iterations in Fig.1 staff) is given below with the solution output in fig. 2.

$$Max\ z = 127104x_1 + 131223x_2 + 135211x_3 + 140421x_4 + 142995x_5 + 159213x_6 + 162372x_7 + 179084x_8 + 180512x_9 + 182750x_{10} + 184152x_{11} + 187289x_{12}$$
$$-74372x_{13} - 76911x_{14} - 80625x_{15} - 83179x_{16} - 88370x_{17} - 91372x_{18} - 94246x_{19} - 96960x_{20} - 99124x_{21} - 102629x_{22} - 113893x_{23} - 118413x_{24}$$

$s.t.$

$$-x_1 + x_{13} \le 370$$
$$-x_1 - x_2 + x_{13} + x_{14} \le 370$$
$$-x_1 - x_2 - x_3 + x_{13} + x_{14} + x_{15} \le 370$$
$$-x_1 - x_2 - x_3 - x_4 + x_{13} + x_{14} + x_{15} + x_{16} \le 370$$
$$-x_1 - x_2 - x_3 - x_4 - x_5 + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} \le 370$$
$$-x_1 - x_2 - x_3 - x_4 - x_5 - x_6 + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} \le 370$$
$$-x_1 - x_2 - x_3 - x_4 - x_5 - x_6 - x_7 + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \le 370$$
$$-x_1 - x_2 - x_3 - x_4 - x_5 - x_6 - x_7 - x_8 + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} \le 370$$
$$-x_1 - x_2 - x_3 - x_4 - x_5 - x_6 - x_7 - x_8 - x_9 + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} + x_{21} \le 370$$
$$-x_1 - x_2 - x_3 - x_4 - x_5 - x_6 - x_7 - x_8 - x_9 - x_{10} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} + x_{21} + x_{22} \le 370$$
$$-x_1 - x_2 - x_3 - x_4 - x_5 - x_6 - x_7 - x_8 - x_9 - x_{10} - x_{11} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} + x_{21} + x_{22} + x_{23} \le 370$$
$$-x_1 - x_2 - x_3 - x_4 - x_5 - x_6 - x_7 - x_8 - x_9 - x_{10} - x_{11} - x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} + x_{21} + x_{22} + x_{23} + x_{24} \le 370$$
$$x_1 \le 230$$
$$x_1 + x_2 - x_{13} \le 230$$
$$x_1 + x_2 + x_3 - x_{13} - x_{14} \le 230$$
$$x_1 + x_2 + x_3 + x_4 - x_{13} - x_{14} - x_{15} \le 230$$
$$x_1 + x_2 + x_3 + x_4 + x_5 - x_{13} - x_{14} - x_{15} - x_{16} \le 230$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 - x_{13} - x_{14} - x_{15} - x_{16} - x_{17} \le 230$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 - x_{13} - x_{14} - x_{15} - x_{16} - x_{17} - x_{18} \le 230$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 - x_{13} - x_{14} - x_{15} - x_{16} - x_{17} - x_{18} - x_{19} \le 230$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 - x_{13} - x_{14} - x_{15} - x_{16} - x_{17} - x_{18} - x_{19} - x_{20} \le 230$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} - x_{13} - x_{14} - x_{15} - x_{16} - x_{17} - x_{18} - x_{19} - x_{20} - x_{21} \le 230$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} - x_{13} - x_{14} - x_{15} - x_{16} - x_{17} - x_{18} - x_{19} - x_{20} - x_{21} - x_{22} \le 230$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} \quad - x_{13} - x_{14} - x_{15} - x_{16} - x_{17} - x_{18} - x_{19} - x_{20} - x_{21} - x_{22} - x_{23} \quad \leq \ 230$$

$$x_1, x_2, \cdots, x_{24} \geq 0$$

**Output**

```
  SIMPLEX METHOD

ITERATION
0
 BASE VAR   VALUE    X1   X2   X3   X4   X5   X6   X7   X8   X9   X10   X11   X12   X13   X14   X15   X16   X17   X18
X19   X20   X21   X22   X23   X24   X25   X26   X27   X28   X29   X30   X31   X32   X33   X34   X35   X36   X37   X38
X39   X40   X41   X42   X43   X44   X45   X46   X47   X48
    X25    370.00 -1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00
    X26    370.00 -1.00 -1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00
    X27    370.00 -1.00 -1.00 -1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 1.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00
    X28    370.00 -1.00 -1.00 -1.00 -1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 1.00 1.00 1.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00
    X29    370.00 -1.00 -1.00 -1.00 -1.00 -1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 1.00 1.00 1.00 1.00 0.00 0.00 0.00
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0.00 0.00 0.00 0.00
    X30    370.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 1.00 1.00 1.00 1.00 1.00 0.00 0.00
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0.00 0.00 0.00 0.00
    X31    370.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 0.00 0.00 0.00 0.00 0.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.00
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0.00 0.00 0.00 0.00
    X32    370.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 0.00 0.00 0.00 0.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
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0.00 0.00 0.00 0.00
    X33    370.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 0.00 0.00 0.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
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0.00 0.00 0.00 0.00
    X34    370.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 0.00 0.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
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0.00 0.00 0.00 0.00
    X35    370.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 0.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
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0.00 0.00 0.00 0.00
    X36    370.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
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0.00 0.00 0.00 0.00
    X37    230.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
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0.00 0.00 0.00
    X38    230.00 1.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 -1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
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0.00 0.00 0.00
    X39    230.00 1.00 1.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 -1.00 -1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
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0.00 0.00 0.00
    X40    230.00 1.00 1.00 1.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 -1.00 -1.00 -1.00 0.00 0.00 0.00 0.00 0.00 0.00
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    X41    230.00 1.00 1.00 1.00 1.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 -1.00 -1.00 -1.00 -1.00 0.00 0.00 0.00 0.00
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    X42    230.00 1.00 1.00 1.00 1.00 1.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 -1.00 -1.00 -1.00 -1.00 -1.00 0.00 0.00 0.00
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0.00 0.00 0.00 0.00
    X43    230.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.00 0.00 0.00 0.00 0.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 0.00 0.00
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0.00 0.00 0.00 0.00
    X44    230.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.00 0.00 0.00 0.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 0.00
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0.00 0.00 0.00 0.00
    X45    230.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.00 0.00 0.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00
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X46 230.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.00 0.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -
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X47 230.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -
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X48 230.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -
1.00 -1.00 -1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 1.00
z 0.0074372.0076911.0080625.0083179.0088370.0091372.0094246.0096960.0099124.00102629.00113893.00118413.00-127104.00-131223.00-135211.00-140421.00-
142995.00-159213.00-162372.00-179084.00-180512.00-182750.00-184152.00-187289.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
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ITERATION
24
BASE VAR VALUE X1 X2 X3 X4 X5 X6 X7 X8 X9 X10 X11 X12 X13 X14 X15 X16 X17 X18
X19 X20 X21 X22 X23 X24 X25 X26 X27 X28 X29 X30 X31 X32 X33 X34 X35 X36 X37 X38
X39 X40 X41 X42 X43 X44 X45 X46 X47 X48
X13 600.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
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X14 600.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
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X15 600.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00
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X16 600.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00
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X17 600.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00
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X18 600.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00
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X19 600.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00
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X20 600.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00
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X22 600.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
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X24 600.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
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X4 600.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
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X5 600.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
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0.00   0.00   0.00
    X6      600.00   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
0.00   0.00   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00   0.00
0.00   0.00   0.00
    X7      600.00   0.00   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00
0.00   0.00   0.00
    X8      600.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   1.00   0.00
0.00   0.00   0.00
    X9      600.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   1.00
0.00   0.00   0.00
    X10      600.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
1.00   0.00   0.00
    X11      600.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
0.00   1.00   0.00
    X12      600.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
0.00   0.00   1.00
    z     502856840.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
0.00
0.0050193.0050598.0052032.0052051.0051623.0064967.0065412.0079960.0077883.0068857.0065739.00187289.0052732.0054312.0054586.0057242.0054625.0067841.006812
6.0082124.0081388.0080121.0070259.0068876.00
  MINIMUM AT z= -
502856840.00
```

**Fig. 1: LP solution**

From the optimal tableau (24th iteration), the optimal solution based on only the decision variables is given as: $x_1 = 230,\ x_2 = x_3 = x_4 = x_5 = x_6 = x_7 = x_8 = x_9 = x_{10} = x_{11} = x_{12} = 600,\ y_1(i.e.\ x_{13}) = y_2(i.e. x_{14}) = y_3(i.e. x_{15}) = y_4(i.e. x_{16}) = y_5(i.e. x_{17}) = y_6(i.e. x_{18}) = y_7(i.e. x_{19}) = y_8(i.e. x_{20}) = y_9(i.e. x_{21}) = y_{10}(i.e. x_{22}) = y_{11}(i.e. x_{23}) = y_{12}(i.e. x_{24}) = 600$ and z=₦502,856,840. From the optimal tableau (24th iteration), the values of the dual variables in two groups are given in Table 3.

**Table 3:** Periodic optimal dual variables **(senior staff)**

| Year | 2001 (1) | 2002 (2) | 2003 (3) | 2004 (4) | 2005 (5) | 2006 (6) | 2007 (7) | 2008 (8) | 2009 (9) | 2010 (10) | 2011 (11) | 2012 (12) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $d_i$ | 50193 | 50598 | 52032 | 52051 | 51623 | 64967 | 65412 | 79960 | 77883 | 68857 | 65739 | 187289 |
| $e_i$ | 52732 | 54312 | 54586 | 57242 | 54625 | 67841 | 68126 | 82124 | 81388 | 80121 | 70259 | 68876 |

$$D_1 = \sum_{i=1}^{12} d_i = 866{,}604 \ and \ E_1 = \sum_{i=1}^{12} e_i = 792{,}232,\ E_1 - D_1 = -74{,}372 \ and \ E_1 + D_1 = 1658836$$

For **post optimality** (i.e. sensitivity) analysis we examine:

(1) Effect of keeping H constant and increasing only h:

The objective function value becomes:

$$z' = z + (H - h)(E_1 - D_1)$$

$$= 502856840 + 370(-74372)$$

= ₦475,339,200

(2) Effect of increasing H by two units and increasing h by one unit:

The objective function value becomes:

$$z' = z + (H - h)(E_1 + D_1)$$

$$= 502856840 + 370(1658836)$$

= ₦1,116,626,160

3(a) Effect of increasing H and keeping h constant:

The objective function value becomes:

$$z' = z + D_1$$

$$= 502856840 + 866604$$

= ₦503,723,444

3(b) Effect of increasing both h and H:

The objective function value becomes:

$$z' = z + E_1$$

$$= 502856840 + 792232$$

= ₦503,649,072

3(c) Effect of reduction in both h and H:

The objective function value becomes:

$$z' = z - E_1$$

$$= 502856840 - 792232$$

= ₦502,064,608

Similar results have been worked out for junior staff and presented in Table 4 which shows the effects of the different type of changes on the objective function value in decreasing order for both junior and senior staff.

**Table 4:** New objective function values for different types of changes

| Types of changes | Objective function value (z') | |
|---|---|---|
| | **Junior staff** | **Senior staff** |
| (2) Increasing H by 2 units and increasing h by 1 unit | ₦37,218,576 | ₦1,116,626,160 |
| 3(a) Increasing H and h is constant | ₦17,726,896 | ₦503,723,444 |
| 3(b) Increasing both h and H | ₦17,696,748 | ₦502,649,072 |
| 3(c) Reducing both h and H | ₦17,642,268 | ₦502,064,608 |
| (1) Increasing h and H constant | ₦10,705,320 | ₦475,339,200 |

**Discussion of Results**

In both the junior and senior staff manpower planning problems of the institution, based on the computer solutions, an increase in the RHS values of the first set of primal constraints always result in greater increase in the objective function value than that of the second set constraints (i.e. $D_1 > E_1$). In the two manpower planning problems (for junior and senior staff) examined in section 4, it is observed that many wastages $(x_j)$ and recruitments $(y_j)$ are equal to the expected capacity $(H)$ of the organization. This means that the dynamic programming and linear programming models are also applicable to manpower planning problems relating to the number of people who are attending workshops, conferences, military courses and skill acquisition training programs that are often organized in batches and sponsored by Governments, NGOs and professional associations as proposed in Teodorescu and Gabriel (2022). This is because all the participants are often recruited and disengaged from such training programs in the same number.

For the sensitivity analysis, Table 4 clearly shows that in both junior and senior staff categories, the objective function value is highest when H is increased by two units and h is increased by one unit. By this type of change, every RHS value of the primal LP problem is increased by only one unit. We must bear in mind that h cannot be increased beyond H which is the target at the end of the time horizon. Each of the remaining four types of changes affect only one set of the RHS values of the primal DP problem. That is why their new objective function values are even lower than half of the highest new objective function value in both junior and senior categories. The lowest of all of them is the case of increasing h while H is unchanged. For this case the new objective function value $(z')$ is lower than the optimal objective function value $(z)$. This sensitivity analysis could not investigate whether the optimality conditions remain satisfied or not for the present optimal solutions in Figs.1 after the different types of changes. This is because some of the cases considered involve increasing h very close to H.

**Conclusion**

This paper extends earlier manpower planning models in Rao (1990) Ogumeyo and Ekoko (2015) and Ogumeyo and Okogun (2023) which deals with determination of periodic optimal recruitment and wastage cost schedule using dynamic programming approach without investigating what happens to the present optimal solution if certain changes occur in the constant parameters of the original problem . This paper has examined the sensitivity analysis of a manpower planning problem, using linear programming and dynamic programming techniques with numerical illustration in which it was observed that the objective function value is highest when H is increased by two units and the initial number of staff (h) is increased by one unity. The sensitivity analysis revealed that some changes in the initial and full capacity manpower in the given numerical illustration, lead to more accruable revenue to the institution than other changes.

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