The effects of inspection errors on single and double inspection sampling plans

Ikpotokin O.*, Braimah O. J, Ehilen O. S. and Edokpa I. W.

Department of Mathematics and Statistics, Faculty of Physical Sciences, Ambrose Alli University, Ekpoma, Nigeria.
*Corresponding author. E-mail: ikpotokinosayomore@yahoo.co.uk.

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In sampling inspection, an inspector is subjected to make type I and type II errors which are unavoidable but can be minimized. This paper therefore investigates the effect of these errors considering Single and Double Sampling plans. The Average Outgoing Quality (AOQ) and Average Total Inspection (ATI) are used to measure the impact of the errors considered as anticipated. In order to measure the impact of these errors by varying the Producer's and Consumer's risk with true fraction of defectives (p) from 0.01 to 0.25, it was observed that as the two risk increases, the values of AOQ decrease for $p < 0.07$; while there was no significant pattern for AOQ for $p = 0.07$ to 0.09. Furthermore, when $p \geq 0.1$, the AOQ increases but the ATI increases uniformly. The same trend was also noticed with the Double Sampling plan but with a bit of increase in values around 1% when compared with the Single Sampling plan.

Key words: Probability of acceptance, defectives items, specification, non-conforming, quality standard, sample lot.

INTRODUCTION

Quality control (QC) is a process by which entities review the quality of all factors involved in production. Controls include product inspection, where every product is examined visually and often using a stereo microscope to find detail before the product is sold into the external market (Braimah and Osanaiye, 2015). Inspectors will be provided with lists and descriptions of unacceptable product defects such as cracks or surface blemishes, for example. Quality control separates the act of testing products to uncover defects from the decision to either allow (acceptance) or deny (reject) product release, which may be determined by fiscal constraints (Schwartz, 1957; Maghsoodloo and Bush, 1985).

Acceptance sampling uses statistical sampling to determine whether to accept or reject a production lot of material. Most often a producer supplies a consumer a number of items and a decision to accept or reject the items is made by determining the number of defective items in a sample from the lot. The lot is accepted if the number of defects falls below the acceptance number or otherwise the lot is rejected (Braimah et al., 2015). Similarly, acceptance sampling is a process of evaluating a portion of production or material from the whole lot for either accepting or rejecting the material on the basis of whether it conforms or does not conform to the set quality standard. A sample is drawn from each lot for inspection; if the amount of defective is less than the prescribed minimum, the lot is accepted. Sampling plans should be designed in such a way that the resulting data will contain a representative sample of the parameters of interest and allow for all questions, as stated in the goals, to be answered (Montgomery, 2009). Acceptance-sampling plans can be classified into quality characteristics that are measured on a numerical scale (variables) and those that are expressed on a "go, no-go" basis (attributes). This paper deals with single and double acceptance-sampling plans for attributes.

A single sampling plan is a procedure by which a single sample is drawn form a lot and inspected. The lot is accepted if the number of nonconforming units found in the sample is less than or equal to the acceptance number (or a specified limit); otherwise, the lot is rejected (Schilling, 1982). While double sampling plan is an extension of a single sampling plan. In double sampling, a second sample is required to decide whether a lot should be rejected or not if the information obtained from the first sample fell into a "gray" area. The procedure for double sampling is to draw and inspect a random sample of size n_1 units from the lot. If the number of non-conforming units, D_1, found in this first sample, is less than or equal to C_1, the lot is accepted. Otherwise the lot will be rejected if D_1 is greater than C_2; a second sample of n_2 units will be taken from the lot and inspected. If the number of non-conforming units from both samples ($D_1 + D_2$) is less than or equal to the acceptance level, C_2, the lot is accepted; otherwise the lot is rejected due to inspection error.

Despite the small amount of reported work considering the effects of human error on inspection plans and procedures, enough exist to point out the need for this research. Jackson (1957) studied the effect of inspector errors on waste and quality control related inspector errors to outgoing quality. An inspector can make two types of errors, which is rejecting an item of acceptable quality or passing an item that is defective. These differ slightly from the customary type I and type II errors usually associated with quality control in that the latter are functions of the sample size and the variability incurred in the sampling program. Inspection errors, however, will occur regardless of whether a sampling plan or 100% inspection is used (Schwartz, 1957). Other considerations of two types of inspector error have been found in the reported studies of Quesenberry (1964). Livingston (1961) presented a study of the efficiency of one hundred percent (100%) inspection based upon the assumption of two types of inspector error. Quesenberry, assuming two types of inspector error, developed an approach to make inferences about the parent population from the results of sampling inspection. These studies have not, however, determined exactly what happens to the AOQ and ATI when two types of inspector error are present, hence this research work.

Adopting Freeman et al. (1948)'s notation, if p_1 represents the probability of a non-defective being misclassified as a defective, and p_2 represents the probability of a defective being misclassified as non-defective; on the average, the inspector will determine the fraction defective to be $p_1(1-p)+p(1-p_2)$, where p is the actual fraction defective. Wetherill and Campling (1966), however, assume that the probability of misclassifying a non-defective as defective is so small as to be negligible. The study considered the effects of a type II Inspector error as rejecting an item of acceptable quality or passing an item that is defective. Using the notation explained in the last section, if e_1 goes to zero, the Inspector, on the average, will classify $p_1(1-e_2)$ items as defective (Jacobson, 1964).

Due to the presence of fraction defectives in inspection process (type II Inspector error), this study aims to rectify the inspection process that gives rooms to rejecting an item of acceptable quality or passing an item that is of defective quality. This paper aims to evaluate the changes in four parameters, which are: producer's risk (α), consumer's risk (β), average outgoing quality (AOQ), and average total inspection (ATI) due to two types of inspection error. The two types of sampling inspection plans to be considered are the single sample and double sample. It will also attempt to derive each of the two types of sampling plans for rectifying inspection.

Rectifying single sample inspection plan

The single sample inspection plan is the least complicated inspection method short of 100% inspection (Jamkhaneh et al., 2011). A sample of predetermined size is drawn from the lot in question. If the sample contains acceptance number (c) or less defective, the lot is accepted; the lot is rejected if more than c defectives are found. The true fraction of defectives (p) may be defined as the proportion of incoming items which is actually defective when inspector error is considered; however, the fraction defective used in any computations is dependent upon this inspector error. It is termed the observed fraction defective (p_e). The formula for p is similar to the one shown by Freeman et al. (1948). It consists of the proportion of good items which will be misclassified as bad items, $e_1(1-p)$ and the proportion of bad items which will be correctly classified as bad items, $p(1-e_2)$. Thus, p_e is the fraction of incoming items which will be judged defective by the inspector and is denoted as:

$$p_e = e_1(1-p) + p(1-e_2) \tag{1}$$

The probability of acceptance is synonymous with the probability of finding acceptance number (c) or less defectives in the sample. Under perfect inspection, the probability of acceptance (Pa) may be estimated from Equation 1 as:

$$P_a = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \tag{2}$$

where c is the acceptance number, i is the dummy variable of summation, n is the sample size, $\binom{n}{i}$ is the number of combinations of n items taken at i time and p is the true fraction defective. The consideration of inspector error in Equation 2 causes the value of the true fraction defective (p) to be substituted by the value of the observed fraction defective p_e and may be expressed as:

$$P_{ae} = \sum_{i=0}^{c} \binom{n}{i} [\mathrm{e}_1(1-\mathrm{p}) + \mathrm{p}(1-\mathrm{e}_2)]^i . [(1 - \mathrm{e}_1(1-\mathrm{p}) - \mathrm{p}(1-\mathrm{e}_2)]^{n-i}$$

or

$$P_{ae} = \sum_{i=0}^{c} \binom{n}{i} {p_e}^i (1-p_e)^{n-i} \tag{3}$$

The average outgoing quality (AOQ) is defined as the proportion of defectives remaining in the lot following the consideration of the lot by the inspector. The expected number of defective items remaining in the lot when it is accepted is denoted by $p(N-n)p_a$. Thus, the average outgoing quality with replacement may be represented by:

$$AOQ = \frac{p(N-n)p_a}{N} \tag{4}$$

Considering inspector error, the actual average outgoing quality with replacement of all items classified as defective may be denoted as:

$$AOQ_e = \frac{np\,e_2 + p(N-n)(1-p_e)p_{ae} + p(N-n)(1-p_{ae})e_2}{N - np_e - (1-p_{ae})(N-n)p_e} \tag{5}$$

where P_{ae} is the probability of acceptance, considering inspector error in Equation 3, p_e is the observed fraction defective in Equation 3; and e_2 is the proportion of bad items misclassified as good. The average amount of inspection, using a single sample inspection plan, depends upon the incoming quality. Assuming perfect inspection, the average amount of inspection may be calculated from:

$$ATI = n + (1-P_a)(N-n) \tag{6}$$

The sample size (n) is always inspected, and the rest of the lot $(N-n)$ is inspected when the lot is rejected with a probability of $(1-P_a)$. When inspector error is considered, the value for the probability of acceptance changes from P_a to P_{ae}. Then, the average amount of inspection may be computed from:

$$ATI_e = n + (1-P_{ae})(N-n) \tag{7}$$

Rectifying double sample inspection plan

Double sample inspection plans offer a possibility of two chances to make a decision on whether to accept or reject the lot in question. A sample of size n_1 is drawn from the lot. Acceptance of the lot on the first sample occurs if the number of defectives found is equal to or less than c_1; rejection occurs if the number of defectives found is greater than c_2. No decision can be made if the number of defectives found in the first sample is greater than c_1 but less than or equal to c_2. Under such circumstances, a second sample of size n_2 is drawn. If the total number of defectives found in both the first and second samples $(n_1 + n_2)$ is less than or equal to c_3, where $c_2 = c_3$. The lot is accepted; otherwise the lot is rejected. The fraction defective in a double sample inspection plan is p_e when perfect inspection is assumed. The apparent fraction defective under inspector error may be represented by,

$$p_e = e_1(1-p) + p(1-e_2) \tag{8}$$

When computing the probability of acceptance for a double sample inspection plan, the possibility of using either one sample or two samples must be taken into account. The probability of acceptance for the perfect inspection plan is equivalent to the probability of finding c_1, or less defective in the first sample; plus the probability of finding c_2 or less in both samples, provided there was no decision made on the first sample. The probability of acceptance on the first sample (p_{a1}) may be expressed by:

$$p_{a1} = \sum_{i=0}^{c_1} \binom{n}{i} p^i (1-p)^{n-i} \tag{9}$$

The probability of acceptance on the second sample (p_{a2}) may be computed by combining the probabilities of the following mutually exclusive conditions. That is $p_{a2} = Pr(x_1 + x_2 \le c_2)$, given $Pr(c_1 < x_1 \le c_2)$, at any given p, where x_1 and x_2 represent the number of defectives found in n_1 and n_2 respectively. Therefore, under perfect inspection, the probability of acceptance on the basis of the second sample may be written as:

$$p_{a2} = \sum_{j=c_1+1}^{c_2} [\binom{n_1}{j} p^j (1-p)^{n_1-j} \sum_{k=0}^{c_2-j} \binom{n_2}{k} p^k (1-p)^{n_2-k}] \tag{10}$$

The probability of acceptance for the combined samples (p_a) is the sum of p_{a1} and p_{a2}, such that $P_a = p_{a1} + p_{a2}$ is

$$P_a = \sum_{i=0}^{c_1} \binom{n}{i} p^i (1-p)^{n-i} + \sum_{j=c_1+1}^{c_2} [\binom{n_1}{j} p^j (1-p)^{n_1-j} \sum_{k=0}^{c_2-j} \binom{n_2}{k} p^k (1-p)^{n_2-k}] \tag{11}$$

To consider inspector error, essentially the same changes must be made as in single sampling by replacing p with p_e. The actual probability of acceptance (P_{ae}) may be denoted by:

$$P_{ae} = \sum_{i=0}^{c_1} \binom{n}{i} {p_e}^i (1-p_e)^{n-i} + \sum_{j=c_1+1}^{c_2} [\binom{n_1}{j} {p_e}^j (1-p_e)^{n_1-j} \sum_{k=0}^{c_2-j} \binom{n_2}{k} {p_e}^k (1-p_e)^{n_2-k}] \tag{12}$$

The probability of rejection on the first sample (P_{r1}) is also of interest when using double sampling plans. This is the probability of finding more than c_2 defective items in the first sample (n_1). Another approach is to observe that if c_2 or less defective is found, the lot is not rejected; therefore, the probability of rejection on the first sample may be represented as:

$$P_{r1} = 1 - \sum_{i=c_1+1}^{c_2} \binom{n_1}{i} p^i (1-p)^{n_1-i} \tag{13}$$

The actual probability of rejection on the first sample, considering inspector error may be denoted by:

$$P_{r1e} = 1 - \sum_{i=c_1+1}^{c_2} \binom{n_1}{i} {p_e}^i (1-p_e)^{n_1-i} \tag{14}$$

The average outgoing quality (AOQ) for double sample inspection plans is calculated as the proportion of defectives remaining in the lot following consideration of the lot by the inspector. The expected number of defectives remaining in the outgoing lot under perfect inspection is the sum of the following groups of items:

(i) The expected number of defectives remaining in the lot when it is accepted on the basis of the first sample $(pp_{a1}(N - n_1)$.
(ii) The expected number of defectives remaining in the lot when it is accepted on the basis of the second sample $(pp_{a2}(N - n_1 - n_2)$.

When inspector error is considered, the expected number of defectives remaining in the lot consists of the following quantities:

(i) the expected number of defective items remaining in the first sample $(n_1 e_2 p)$
(ii) the expected number of defective items remaining in the second sample which is inspected if no decision is made on the first sample $(n_2(1 - p_{a1e})e_2 p)$
(iii) the expected number of defective items remaining in the lot if it is accepted based on the first sample $(N - n)p_{a1e}p$;
(iv) the expected number of defective items remaining in the lot if it is not accepted on the first sample but is accepted on the second sample $((N - n_1 - n_2)\ p_{a2e}p)$
(v) the expected number of defective items remaining in the lot after the one hundred percent inspection of the lot is rejected on the basis of the first sample, $((N - n_1)\ p_{r1e} e_2\ p)$
(vi) the expected number of defective items remaining in the lot after the one hundred percent inspection of the lot is not rejected based on the first sample but is rejected after the second sample $((N - n_1 - n_2)\ (1 - p_{ae} - p_{r1e})\ e_2\ p\ )$.
The average outgoing quality (AOQ) after considering the inspector error is the sum of these quantities divided by N (the total number of items passed from the inspection procedure); since all items classified as defectives are replaced by non-defectives

$$AOQ_e = \left(\frac{[n_1 e_2 p + n_2(1 - p_{d1e})e_2 p + (N-n)p_{a1e}p + (N-n_1-n_2)p_{a2e}p + (N-n_1)\ p_{r1e}e_2\ p + (N-n_1-n_2)\ (1-p_{ae} - p_{r1e})\ e_2\ p]}{N}\right) \quad (16)$$

The average total inspection (ATI) associated with a double sample inspection plan is the sum of the following groups of items when perfect inspection is assumed:

(i) The first sample which is always inspected (n_1)
(ii) The expected number of items when no decision is made on the first sample $(n_2(1 - p_{a1}))$
(iii) The expected number of items when the lot is rejected on the first sample $(\ (N - n_1)\ p_{r1})$
(iv) The expected number of items if no decision is made on the second sample, $(N - n_1 - n_2)\ (1 - p_a - p_{r1})$.

Under perfect inspection when inspector error is considered, the average total inspection (ATI) for double sample may be calculated above from the preceding terms in reduced form as,

$$ATI_e = n_1 + n_2(1 - p_{a1e}) + (N - n_1 - n_2)(1 - p_{ae}) \quad (17)$$

METHODOLOGY

The methods are based on the assumption that the inspector is incapable of error. An inspector is capable of making two types of error; a type I inspector error is that of classifying a good item as bad, and a type II inspector error is that of classifying a bad item as good. The expected proportion of non-defective items misclassified as defective items is denoted by e_1 and the expected proportion of defective items misclassified as non-defective is denoted by e_2 (Juran, 1999). The methodologies used in this study are the producer's risk (α), consumer's risk (β), average outgoing quality (AOQ), and average total inspection (ATI) due to two types of inspection error e_1 and e_2. The two types of sampling inspection plans that will be considered are the single and double sampling plans. In order to facilitate computational efficiency, simulation was carried out in Microsoft Excel 2007 by varying the values of both e_1 and e_2 on one hand, varying e_1 and holding e_2 constant or vis-a-vis on the other hand and doing appropriate analyses for individual output.

RESULTS AND DISCUSSION

This section presents the result simulated with Microsoft Excel 2007 and appropriate discussion for individual output.

Results for single sampling plan

Effect of increase in both producer's risks (α) and consumer's risks (β) with strict reference to AOQ is shown in Table 1. Figure 1 shows that as the two risks increase, the values of AOQ decrease from p < 0.07. There is no pattern for AOQ from p = 0.07 to 0.09; while p ≥ 0.1 AOQ increases. Table 2 shows the effect of varying consumer's risk (β) and making constant producer's risk (α) with reference to AOQ; Figure 2 shows that as AOQ increases, the consumer's risk (β) increases when other factors are fixed. Table 3 shows the effect of varying producer's

Table 1. Effect of increase in α, β on average outgoing quality (AOQ).

e_1	0.05	0.04	0.03	0.02	0.01	0
e_2	0.05	0.04	0.03	0.02	0.01	0
P	AOQ_1	AOQ_2	AOQ_3	AOQ_4	AOQ_5	AOQ_6
0.01	0.00296	0.00422	0.00576	0.00731	0.00843	0.00889
0.02	0.00413	0.00591	0.00843	0.01151	0.01457	0.01679
0.03	0.00436	0.00603	0.00871	0.01247	0.01703	0.02159
0.04	0.00427	0.00547	0.00769	0.01121	0.01613	0.02212
0.05	0.00418	0.00482	0.00630	0.00899	0.01326	0.01922
0.06	0.00425	0.00435	0.00508	0.00678	0.00985	0.01471
0.07	0.00448	0.00416	0.00425	0.00502	0.00684	0.01017
0.08	0.00486	0.00420	0.00381	0.00385	0.00461	0.00646
0.09	0.00534	0.00443	0.00367	0.00319	0.00315	0.00382
0.1	0.00590	0.00478	0.00376	0.00289	0.00229	0.00213
0.15	0.00920	0.00730	0.00544	0.00360	0.00181	0.00006
0.2	0.01299	0.01031	0.00767	0.00508	0.00252	0.00000
0.25	0.01724	0.01370	0.01020	0.00676	0.00336	0.00000

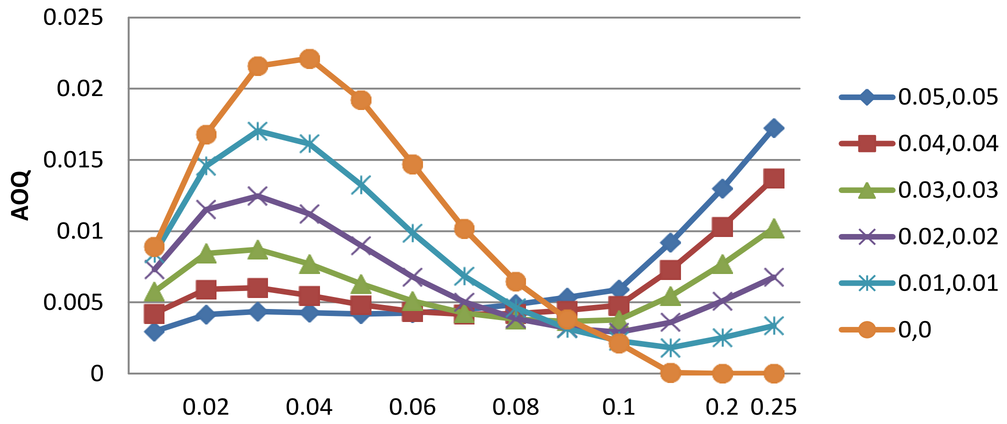

Figure 1. Apparent AOQ versus fraction defective.

risk (α) and making constant consumer's risk (β) with reference to AOQ. Figure 3 shows that as producer's risk increases the AOQ decreases from p = 0.01 to 0.1; while the later increases at p ≥ 0.15. Effect of increase in both producer's risks (α) and consumer's risks (β) with strict reference to Average Total Inspection (ATI) is shown in Table 4, while Figure 4 shows that as the two risks increase, the values of ATI increase uniformly. Also, effect of varying consumer's risk (β) and making constant producer's risk (α) with reference to ATI is shown in Table 5, and Figure 5 shows that ATI decreases as the consumer's risk increases when

Table 2. Effect of varying Consumer's risk (β) on AOQ.

e_1	**0.05**	**0.05**	**0.05**	**0.05**	**0.05**
e_2	0.04	0.03	0.02	0.01	0
P	AOQ_1	AOQ_2	AOQ_3	AOQ_4	AOQ_5
0.01	0.00287	0.00278	0.00269	0.00260	0.00251
0.02	0.00392	0.00371	0.00350	0.00328	0.00307
0.03	0.00402	0.00368	0.00334	0.00300	0.00265
0.04	0.00380	0.00334	0.00287	0.00241	0.00194
0.05	0.00360	0.00302	0.00244	0.00185	0.00127
0.06	0.00355	0.00286	0.00216	0.00146	0.00077
0.07	0.00367	0.00286	0.00205	0.00125	0.00044
0.08	0.00394	0.00301	0.00209	0.00116	0.00023
0.09	0.00430	0.00326	0.00221	0.00117	0.00012
0.1	0.00474	0.00357	0.00240	0.00123	0.00006
0.15	0.00738	0.00554	0.00370	0.00186	0.00000
0.2	0.01042	0.00783	0.00524	0.00262	0.00000
0.25	0.01384	0.01042	0.00697	0.00350	0.00000

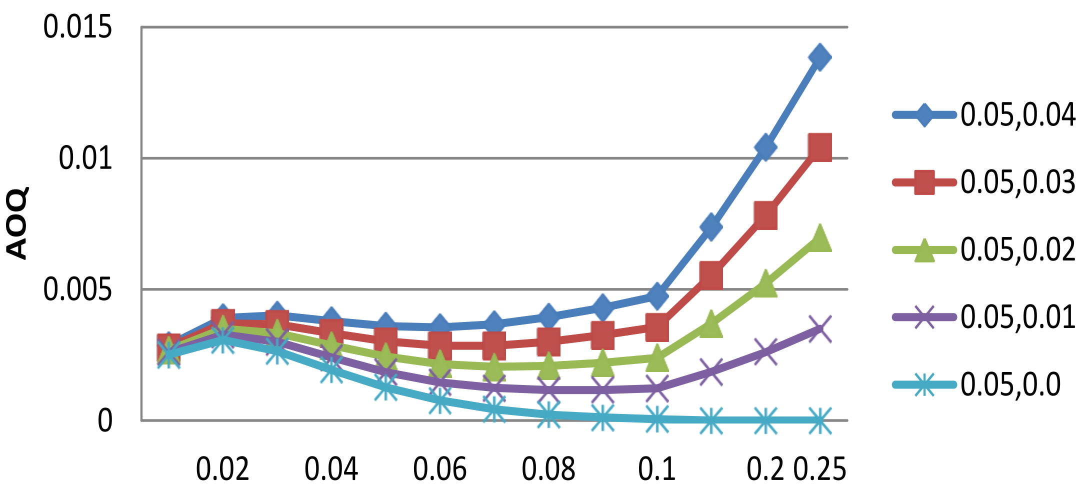

Figure 2. Apparent AOQ versus fraction defective.

Table 3. Effect of varying producer's risk (α) on AOQ.

e_1	**0.05**	**0.04**	**0.02**	**0.01**	**0**
e_2	0.03	0.03	0.03	0.03	0.03
P	AOQ_1	AOQ_2	AOQ_3	AOQ_4	AOQ_5
0.01	0.00278	0.00414	0.00735	0.00847	0.00893
0.02	0.00371	0.00571	0.01166	0.01478	0.01698
0.03	0.00368	0.00569	0.01279	0.01759	0.02222
0.04	0.00334	0.00499	0.01171	0.01712	0.02346
0.05	0.00302	0.00422	0.00965	0.01463	0.02132
0.06	0.00286	0.00364	0.00756	0.01153	0.01741
0.07	0.00286	0.00334	0.00591	0.00874	0.01327
0.08	0.00301	0.00327	0.00483	0.00667	0.00981
0.09	0.00326	0.00339	0.00426	0.00537	0.00736
0.1	0.00357	0.00362	0.00406	0.00468	0.00586
0.15	0.00554	0.00549	0.00539	0.00536	0.00535
0.2	0.00783	0.00775	0.00760	0.00752	0.00745
0.25	0.01042	0.01031	0.01010	0.01000	0.00990

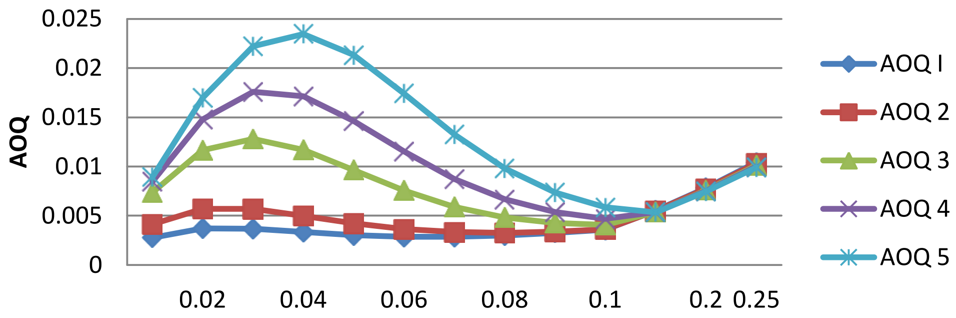

Figure 3. Apparent AOQ versus fraction defective.

Table 4. Effect of increase in α, β on ATI.

e_1	0.05	0.04	0.03	0.02	0.01	0
e_2	0.05	0.04	0.03	0.02	0.01	0
P	ATI_1	ATI_2	ATI_3	ATI_4	ATI_5	ATI_6
0.01	738.39719	594.55370	423.26042	257.89575	144.17318	103.08909
0.02	835.90757	730.69164	587.94791	419.67637	257.89575	145.74740
0.03	902.33228	832.27444	728.08948	587.94791	423.26042	263.93067
0.04	944.46207	901.14841	832.27444	730.69164	594.55370	434.02234
0.05	969.66443	944.46207	902.33228	835.90757	738.39719	607.61683
0.06	984.01534	970.08202	945.89060	905.81218	842.98803	750.90230
0.07	991.84670	984.47616	971.30364	948.65052	911.37809	853.15218
0.08	995.96318	992.21299	985.36112	973.23922	952.55645	918.69710
0.09	998.05557	996.21286	992.78937	986.60123	975.74934	957.35199
0.1	999.08720	998.21006	996.56012	993.52928	988.10327	978.66003
0.15	999.98522	999.97093	999.94354	999.89172	999.79500	999.61704
0.2	999.99986	999.99972	999.99948	999.99903	999.99819	999.99667
0.25	1000.00000	1000.00000	1000.00000	999.99999	999.99999	999.99998

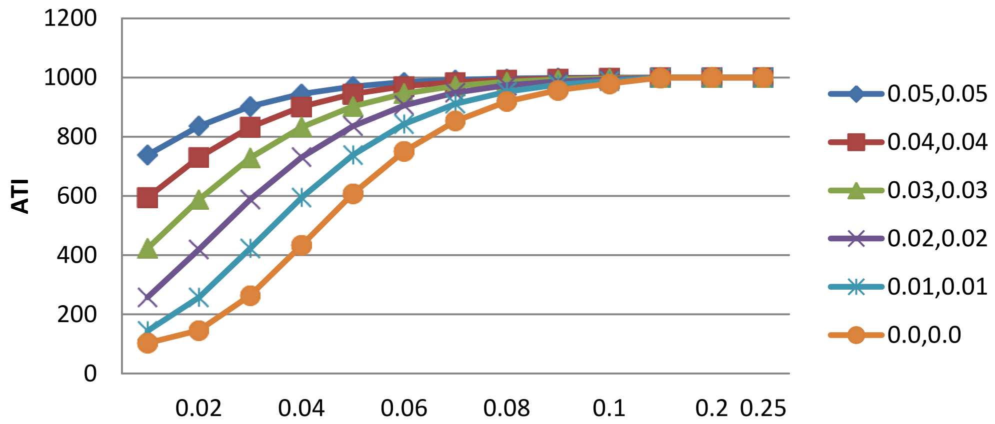

Figure 4. Apparent ATI versus fraction defective.

other factors are fixed. Table 6 shows the effect of varying producer's risk (α) and making constant consumer's risk (β) with reference to ATI. Figure 6 shows that as producer's risk increases while consumer's risk is kept constant, ATI increases.

Results for double sampling plan

Table 7 shows the effect of increase in both producer's and consumer's risks (α and β) with strict reference to AOQ. Figure 7 shows that as the two risks increase, the values of AOQ decrease from p < 0.07. There is no pattern for

Table 5. Effect of varying Consumer's risk (β) on ATI.

e_1	0.05	0.05	0.05	0.05	0.05
e_2	0.04	0.03	0.02	0.01	0
P	ATI_1	ATI_2	ATI_3	ATI_4	ATI_5
0.01	739.66670	740.93200	742.19307	743.44992	744.70255
0.02	837.70082	839.47860	841.24098	842.98803	844.71981
0.03	904.08559	905.81218	907.51231	909.18630	910.83442
0.04	945.89060	947.28656	948.65052	949.98311	951.28489
0.05	970.69861	971.70065	972.67142	973.61175	974.52249
0.06	984.70192	985.36112	985.99390	986.60123	987.18403
0.07	992.27252	992.67740	993.06230	993.42813	993.77577
0.08	996.21286	996.44787	996.66901	996.87704	997.07271
0.09	998.19515	998.32514	998.44615	998.55879	998.66360
0.1	999.16203	999.23096	999.29443	999.35285	999.40661
0.15	999.98723	999.98898	999.99049	999.99180	999.99293
0.2	999.99988	999.99991	999.99992	999.99994	999.99995
0.25	1000.00000	1000.00000	1000.00000	1000.00000	1000.00000

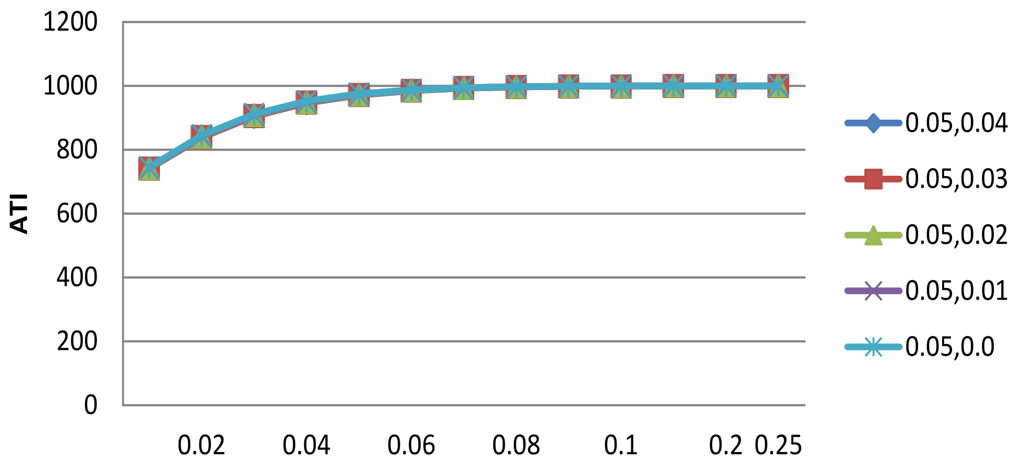

Figure 5. Apparent ATI versus fraction defective.

Table 6. Effect of varying Producer's risk (α) on ATI.

e_1	0.05	0.04	0.02	0.01	0
e_2	0.03	0.03	0.03	0.03	0.03
P	ATI_1	ATI_2	ATI_3	ATI_4	ATI_5
0.01	740.93200	596.19755	256.40084	142.63171	102.71542
0.02	839.47860	733.27699	416.09459	251.95006	141.12293
0.03	905.81218	835.00513	582.96233	412.51547	250.47794
0.04	947.28656	903.50414	725.47054	581.29464	412.51547
0.05	971.70065	946.24262	831.35639	725.47054	582.96233
0.06	985.36112	971.30364	902.33228	832.27444	728.08948
0.07	992.67740	985.25311	946.24262	903.50414	835.00513
0.08	996.44787	992.67740	971.70065	947.28656	905.81218
0.09	998.32514	996.47625	985.68075	972.47973	948.98659
0.1	999.23096	998.35278	993.00851	986.19913	973.61175
0.15	999.98898	999.97483	999.87571	999.73159	999.43185
0.2	999.99991	999.99978	999.99880	999.99728	999.99391
0.25	1000.00000	1000.00000	999.99999	999.99998	999.99996

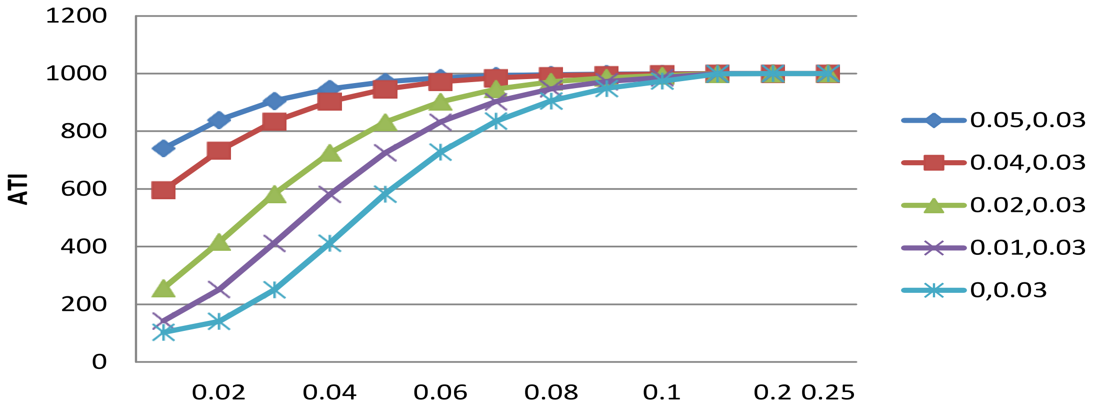

Figure 6. Apparent ATI versus fraction defective.

Table 7. Effect of increase in α, β on AOQ.

e_1	0.05	0.04	0.03	0.02	0.01	0
e_2	0.05	0.04	0.03	0.02	0.01	0
P	AOQ_1	AOQ_2	AOQ_3	AOQ_4	AOQ_5	AOQ_6
0.01	0.00307	0.00450	0.00628	0.00791	0.00881	0.00899
0.02	0.01242	0.01442	0.01618	0.01757	0.01834	0.01830
0.03	0.01559	0.01863	0.02152	0.02410	0.02620	0.02743
0.04	0.00412	0.00543	0.00784	0.01173	0.01732	0.02418
0.05	0.00394	0.00468	0.00628	0.00920	0.01389	0.02065
0.06	0.00391	0.00413	0.00496	0.00679	0.01010	0.01543
0.07	0.00404	0.00384	0.00405	0.00494	0.00690	0.01045
0.08	0.00431	0.00380	0.00354	0.00370	0.00457	0.00656
0.09	0.00467	0.00393	0.00333	0.00298	0.00306	0.00385
0.1	0.00509	0.00417	0.00333	0.00263	0.00218	0.00214
0.15	0.00750	0.00600	0.00451	0.00302	0.00153	0.00006
0.2	0.01000	0.00800	0.00600	0.00400	0.00200	0.00000
0.25	0.01250	0.01000	0.00750	0.00500	0.00250	0.00000

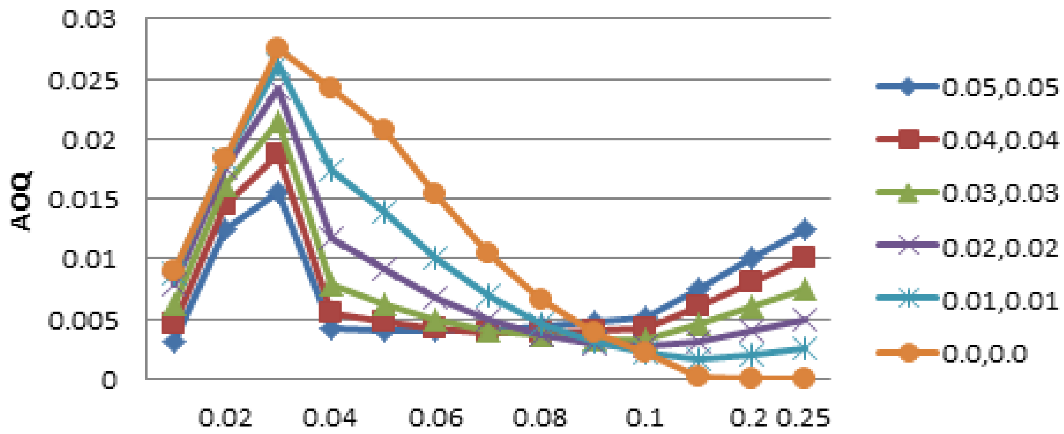

Figure 7. Apparent AOQ versus fraction defective.

AOQ from p = 0.07 to 0.09; while p ≥ 0.1 AOQ increases. Table 8 shows the effect of varying consumer's risk (β) and making constant producer's risk (α) with reference to AOQ, and Figure 8 shows that as AOQ increases, the consumer's risk (β) increases when other

Table 8. Effect of varying consumer's risk (β) on AOQ.

e_1	0.05	0.05	0.05	0.05	0.05
e_2	0.04	0.03	0.02	0.01	0
P	AOQ_1	AOQ_2	AOQ_3	AOQ_4	AOQ_5
0.01	0.00298	0.00290	0.00281	0.00272	0.00264
0.02	0.01229	0.01217	0.01204	0.01191	0.01178
0.03	0.01533	0.01508	0.01482	0.01456	0.01429
0.04	0.00369	0.00325	0.00282	0.00239	0.00196
0.05	0.00341	0.00288	0.00234	0.00181	0.00128
0.06	0.00328	0.00265	0.00202	0.00140	0.00077
0.07	0.00332	0.00260	0.00188	0.00116	0.00044
0.08	0.00349	0.00268	0.00186	0.00105	0.00023
0.09	0.00376	0.00285	0.00194	0.00103	0.00012
0.1	0.00408	0.00307	0.00207	0.00106	0.00006
0.15	0.00600	0.00450	0.00300	0.00150	0.00000
0.2	0.00800	0.00600	0.00400	0.00200	0.00000
0.25	0.01000	0.00750	0.00500	0.00250	0.00000

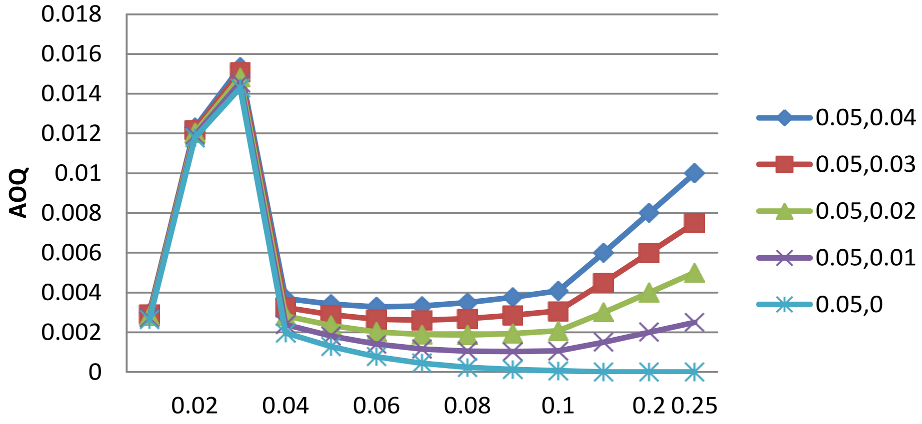

Figure 8. Apparent AOQ versus fraction defective.

factors are fixed.

Table 9 shows the effect of varying producer's risk (α) and making constant consumer's risk (β) with reference to AOQ. Figure 9 also shows that as producer's risk increases the AOQ decreases from p = 0.01 to 0.1; while AOQ remains fairly constant at p = 0.15 to 0.25. Table 10 shows the effect of increase in both producer's and consumer's risks (α and β) with strict reference to ATI; while in Figure 10, as the two risks increase, the values of ATI increase uniformly. Table 11 shows the effect of varying consumer's risk (β) and making constant producer's risk (α) with reference to ATI. However, Figure 11 shows that ATI decreases as the consumer's risk increases when other factors are fixed; but at p = 0.02 there is an irregular pattern. As p = 0.15 to 0.25 ATI was fairly constant. Table 12 shows the effect of varying producer's risk (α) and making constant consumer's risk (β) with reference to ATI. Figure 12 shows that as producer's risk increases while consumer's risk is kept constant, ATI increases.

DISCUSSION

In an attempt to increase both the producer's and consumer's risk for single and double sampling plans, the values of AOQ decrease from $p < 0.07$ with no pattern for p = 0.07 to 0.09; while $p \geq 0.1$ AOQ increases. The effect of varying consumer's risk (β) and keeping constant producer's risk shows that as AOQ increases, the consumer's risk (β) increases when other factors are fixed. Effect of varying producer's risk (α) and making

Table 9. Effect of varying producer's risk (α) on AOQ.

e_1	**0.05**	**0.04**	**0.02**	**0.01**	**0**
e_2	0.03	0.03	0.03	0.03	0.03
P	AOQ_1	AOQ_2	AOQ_3	AOQ_4	AOQ_5
0.01	0.00290	0.00443	0.00795	0.00884	0.00902
0.02	0.01217	0.01431	0.01765	0.01843	0.01836
0.03	0.01508	0.01841	0.02423	0.02638	0.02756
0.04	0.00325	0.00498	0.01224	0.01835	0.02556
0.05	0.00288	0.00412	0.00984	0.01530	0.02285
0.06	0.00265	0.00347	0.00754	0.01176	0.01819
0.07	0.00260	0.00310	0.00577	0.00872	0.01352
0.08	0.00268	0.00297	0.00460	0.00651	0.00978
0.09	0.00285	0.00301	0.00395	0.00511	0.00717
0.1	0.00307	0.00316	0.00368	0.00434	0.00556
0.15	0.00450	0.00450	0.00452	0.00454	0.00458
0.2	0.00600	0.00600	0.00600	0.00600	0.00600
0.25	0.00750	0.00750	0.00750	0.00750	0.00750

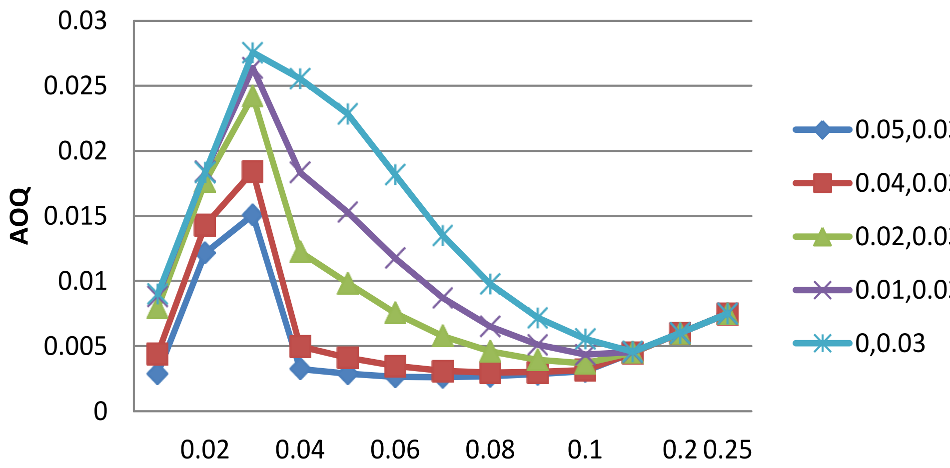

Figure 9. Apparent AOQ versus fraction defective.

Table 10. Effect of increase in α, β on ATI.

e_1	**0.05**	**0.04**	**0.03**	**0.02**	**0.01**	**0**
e_2	0.05	0.04	0.03	0.02	0.01	0
P	ATI_1	ATI_2	ATI_3	ATI_4	ATI_5	ATI_6
0.01	729.46319	572.63779	383.78548	212.77177	120.18401	100.68198
0.02	399.27823	293.36712	200.80456	126.79148	84.82750	85.01820
0.03	505.65107	394.65939	291.28917	200.80456	128.07104	85.70791
0.04	944.19736	900.13940	828.91429	721.18492	572.63779	395.51972
0.05	969.60050	944.19736	901.35087	832.70859	729.46319	587.03999
0.06	984.00133	970.02015	945.64154	904.90947	840.09121	742.86233
0.07	991.84388	984.46308	971.24757	948.43018	910.59376	850.66151
0.08	995.96265	992.21047	985.34975	973.19168	952.37338	918.05453
0.09	998.05548	996.21241	992.78727	986.59203	975.71168	957.20941
0.1	999.08718	998.20998	996.55977	993.52766	988.09634	978.63221
0.15	999.98522	999.97093	999.94354	999.89172	999.79500	999.61704
0.2	999.99986	999.99972	999.99948	999.99903	999.99819	999.99667
0.25	1000.00000	1000.00000	1000.00000	999.99999	999.99999	999.99998

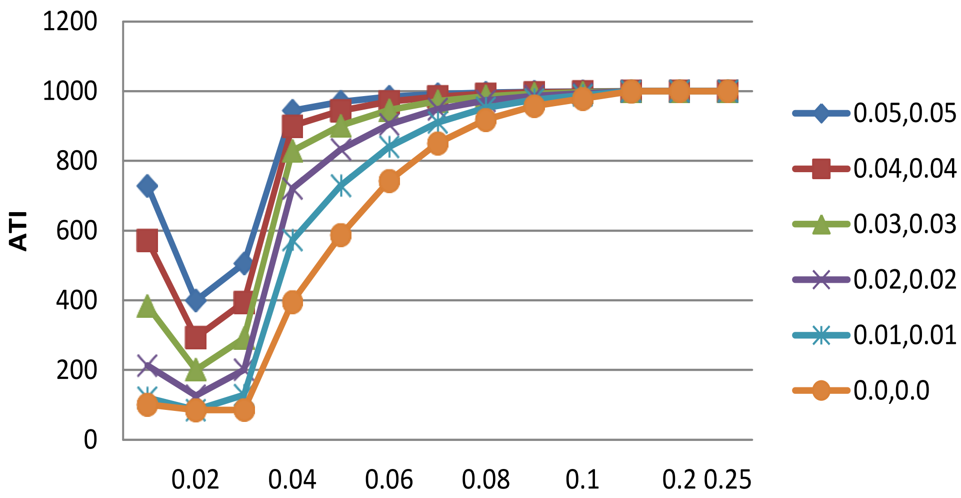

Figure 10. Apparent ATI versus fraction defective.

Table 11. Effect of varying Consumer's risk (β) on ATI.

e_1	0.05	0.05	0.05	0.05	0.05
e_2	0.04	0.03	0.02	0.01	0
P	ATI_1	ATI_2	ATI_3	ATI_4	ATI_5
0.01	730.82548	732.18279	733.53512	734.88249	736.22488
0.02	401.59288	403.91092	406.23227	408.55687	410.88464
0.03	509.22895	512.80561	516.38074	519.95402	523.52513
0.04	945.64154	947.05227	948.43018	949.77591	951.09011
0.05	970.63971	971.64640	972.62146	973.56576	974.48016
0.06	984.68930	985.34975	985.98367	986.59203	987.17575
0.07	992.27004	992.67522	993.06039	993.42645	993.77429
0.08	996.21241	996.44748	996.66868	996.87676	997.07247
0.09	998.19507	998.32507	998.44610	998.55875	998.66356
0.1	999.16202	999.23095	999.29442	999.35284	999.40660
0.15	999.98723	999.98898	999.99049	999.99180	999.99293
0.2	999.99988	999.99991	999.99992	999.99994	999.99995
0.25	1000.00000	1000.00000	1000.00000	1000.00000	1000.00000

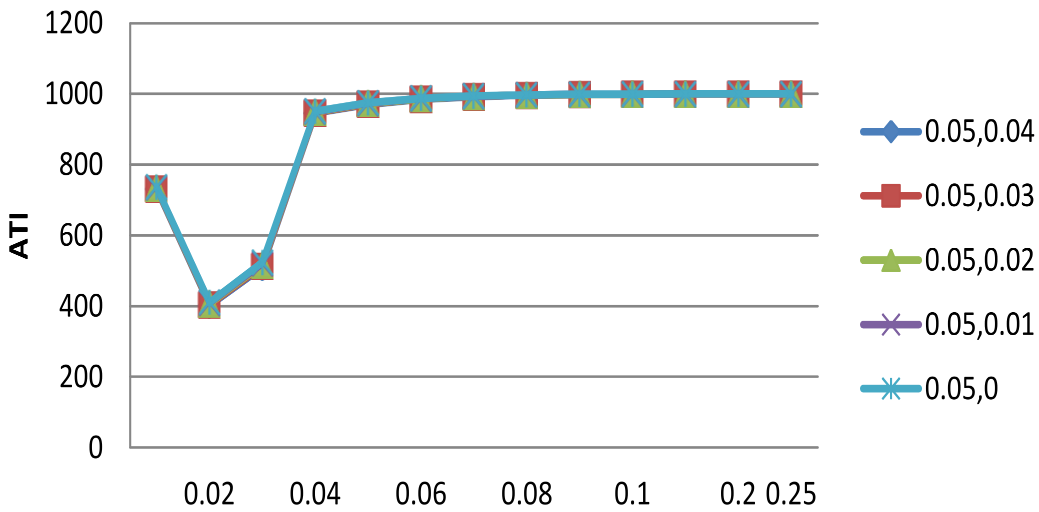

Figure 11. Apparent ATI versus fraction defective.

Table 12. Effect of varying producer's risk (α) on ATI.

e_1	**0.05**	**0.04**	**0.02**	**0.01**	**0**
e_2	0.03	0.03	0.03	0.03	0.03
P	ATI_1	ATI_2	ATI_3	ATI_4	ATI_5
0.01	732.18279	574.45129	211.35845	119.23623	100.58703
0.02	403.91092	295.45094	125.52566	84.02801	85.97332
0.03	512.80561	398.12220	198.16972	124.27378	83.84076
0.04	947.05227	902.54964	715.56650	557.99975	372.11081
0.05	971.64640	945.99734	827.95485	715.56650	559.84184
0.06	985.34975	971.24757	901.35087	828.91429	718.38567
0.07	992.67522	985.24154	945.99734	902.54964	831.76651
0.08	996.44748	992.67522	971.64640	947.05227	904.90947
0.09	998.32507	996.47588	985.66997	972.42893	948.76960
0.1	999.23095	998.35272	993.00656	986.18925	973.56576
0.15	999.98898	999.97483	999.87571	999.73159	999.43184
0.2	999.99991	999.99978	999.99880	999.99728	999.99391
0.25	1000.00000	1000.00000	999.99999	999.99998	999.99996

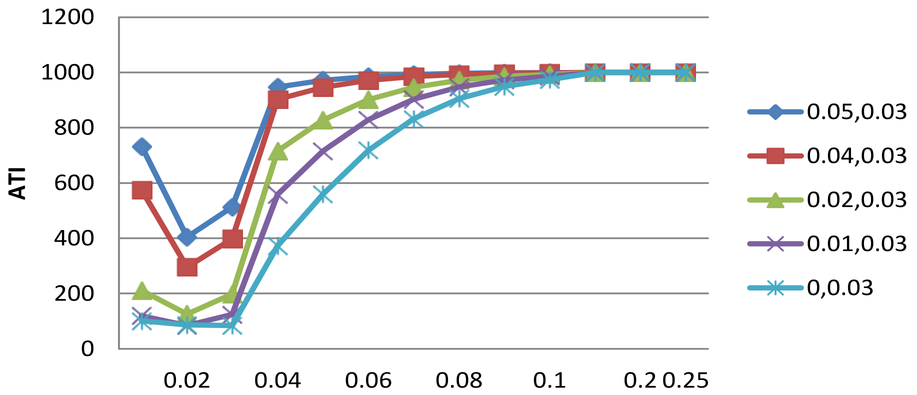

Figure 12. Apparent ATI versus fraction defective.

constant consumer's risk (β) shows that as producer's risk increases the AOQ decreases from p = 0.01 to 0.1; while the AOQ increases at p ≥ 0.15. However, increase in both producer's and consumer's risks (α and β) with reference to ATI shows that as the two risks increase, the values of ATI increase uniformly. Nevertheless, varying consumer's risk (β) and making constant producer's risk (α) shows that ATI decreases as the consumer's risk increases when other factors are fixed. But at p = 0.02, there is an irregular pattern and at p = 0.15 to 0.25 ATI was fairly constant for double sampling. The effect of varying producer's risk (α) and making constant consumer's risk (β) shows that as producer's risk increases while consumer's risk is kept constant, ATI increases.

Conclusion

In conclusion, in order to reduce the effect of sampling errors, the study presented an estimation error for inspections by using a well-designed experiment. In the event where the level of error is high, inspectors should be trained to minimize these errors.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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