A comparative study of three stochastic approximation methods with application on regime switching process

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The clamour for stock Market business has been on the increase in recent times. As a result, there is the urgent need to study certain tools that enhance positive analysis of the market for higher gains. In the light of this, this article focused on the simulation of Runge-Kutta, Milstein and Taylor's methods on Regime Switching Process using MAPLE 18. It equally considered the volatility rate of the stock market by simulating Runge-Kutta, Milstein and Taylor's methods with application on Regime Switching Process. We compared the results in the case of high *volatility* with zero drift and also high drift with low volatility. Our findings showed that RungeKutta method has the best prediction power of volatility in stock market followed by Milstein and Taylor's method on Regime Switching Process. Based on these findings, we recommend the onward adoption and application of Runge-Kutta method by stock investors for the smooth solving of instability problems of stock market especially on regime switching process.

Key words: Volatility, simulation, stock market, regime switching process, zero drift, high drift, instability, stock investors.

INTRODUCTION

Several articles and journals have treated numerical methods for stochastic differential equations, and their optimized results, but the gap between the discussed theory of SDEs and its application is still worrisome. This gap with respect to instability in stock market according to chaos theory reviewed by Halton (2019) can be bridged, since mathematical concepts has explained that the probability of random results of most unstable conditions emanate from equations. This theory has equally predicted the behavioural patterns of the price of goods and services in stock market. In order to the bridge of this gap emanating from these scenarios, the effectiveness of any numerical methods for SDEs must have been simulated severally on the sample paths in order to obtain the desired results by the approximation of statistical analyses using the current computer computing errors (Petersen, 1987). According to Lyuu (2012), an Ito process has an equivalence value with stochastic process X =

$X = \{X_t, t, 0\}$which is produced, (1)

where $t \geq 0$ and X_0 represent time and scalar point of starting respectively.

To satisfy the Stochastic Processes $a(X_t, t)$: $t \geq 0$ and $b(X_t , t)$: $t \geq 0$ certain conditions such as $a(X_t, t)$ and $b(X_t, t)$ which are the drift and diffusion must respectively show that the Ito Stochastic differential equation is

$$dX_t = a((X_t,t)dt + b((X_t,t)dt \quad (2)$$

Where the drift, a_t and variance, b_t^2 both aligned with Brownian motion. Therefore, Brownian motion can be defined as the random movement (diffusion) of particles colliding with several other elements. It has so much influence on the study of different disciplines especially in explaining the volatility problems of the stock market. The most important branch of stochastic process considered Markov chain as it satisfies, the Markov properties known also as Markov process because its outcomes are possibly in the finite state and has constant probabilities over time and its attribute is memory less (Monica et al., 2019). This has great influence in some research work produced in the recent times and have shown significant results, thus, the intensity of bombarding molecules of elements has less effect on state of the variables. But when the noise is due to external force which depends on the nature of system, there is much influence on stock investment and option pricing. Thus, its integral form is`

$$X = X_0 + \int_0^t a(Xs)ds + \int_0^t b(Xs)d\,Ws \quad (3)$$

The apparent unstable random prices of share in stock exchange market uses stochastic differential equation. This was first considered by Merton (1974) and his approach has produced acceptable results to credit risk measures. The relevance of its model assures its function to public trade company especially stock exchange rather than data finance company (Misankova and Kosicova, 2014). Merton (2014) also considered two kinds of investment – one risk free and the other risky. To him, an investor must adopt a strategy which will utilize its function maximally. For example, suppose the price P_s of the risk-free investment has constantly increased with growth proportional to the exponential increase of ordinary differential equation, then

$$P_s = \mathrm{a}p_s \quad (4)$$

Let price P_r of the risky investment, satisfied with similar equation under noisy disturbance be proportional to the price.

$$P_r = \mathrm{b}P_r + \mathrm{B}P_r\xi_t \quad (5)$$

Where ξt represents Gaussians white noise process producing stochastic differential equation.

$$\mathrm{d}P_t{}^r = bP_t{}^r dt + BP_t{}^r dW_t \quad (6)$$

From Equation 5, {W(t), t ≥ 0} represents standard Wiener process, b and B are positive constants where a < b. The investor at each time must choose the fraction f of his wealth which must be invested with risk and the fraction (1 – f) left to be considered as safe one. If his current utility rate is c ≥ 0, then with Equation 4 and 5, his wealth X_t satisfies the stochastic differential equation.

$$\mathrm{d}X_t = \mathrm{f}(\mathrm{b}\ X_t dt + BX_t dW_t) + (1-f)\ \{\mathrm{a}\ X_t dt - cdt\} \quad (7)$$

Which also produces another stochastic differential equation with respect to a disturbance (noisy fluctuation).

$$\mathrm{d}\ X_t = (\{(1-f)a + fb\}X_t - c)dt + fBX_t dW_t \quad (8)$$

According to Valaskova, and Dengov (2014), the risk of losing as a result of defaults on loans to borrowers is seen as credit risk. This results from the inability to comply with the terms and conditions involved in the finance transaction contract. Suppose the risky asset price of an exchange rate of stock develops to the stochastic differential equation.

$$X_t = X_0 + \int_0^t b(s, X_s)dW_s \quad (9)$$

S, X_s are the independent variables (intensity of the noise).

Consider the situation where the interest rate is zero and no dividends then European call such option as striking price C at time T where price C is fixed, thus, we have

$$\mathrm{F}(X_T) = (X_T - C)^+ = \begin{cases} X_T - C, & X_T > C \\ 0, & X_T \le C \end{cases} \quad \forall t \in [0, T] \quad (10)$$

Where w is a Wiener process with probability measure (s, X_s) is the coefficient of diffusion and dW_s is the Gaussian white noise. In financial sector when someone borrows from a lender in attempt to create wealth for self in whatever means the borrowing may be done either through Bank loan or any financial loan institution ,bonds are issued for security purpose . However, Models for current credit risk have been built on trial and error to avoid inefficient theoretical framework .Therefore Banks in this regard have used traditional static modelling framework in the assessment of the credit risk of their customers. This static model has been tested and proven to reasonably work efficiently for specific periods (Moradi, 2019).

The problem of instability in the world economy especially in stock exchange market has been a huge concern to economists, the world economic monitoring team, the accountants etc. However, we intend to modify and simulate Runge-Kutta, Milstein and Taylor's Methods to enable us checkmate the economic system and optimize the approximation results out of the three methods to correct the unstable conditions of stock exchange market. The general objective of this work is to simulate three numerical approximations, Runge-kutta, Milstein and Taylor's methods on Regime Switching Process.

The rest of this paper will be organized in the following manner; section two will present the three simulation methods intended for this study viz: Runge-Kutta, Milstein and Taylor's Method on regime switching process; section three shall focus on the regime switching process taking into cognisance the regime switching process command, the asset pricing regimes and exchange rates as often encountered in the stock exchange market; while section four will look at the resulting effects of volatility and drift parameters through the simulation of the (RKM, T & M) methods. Finally, we draw conclusions on our discussions so far.

The simulation methods

Here, we present the simulation methods intended to assist us in our analysis of the stock market in relation with the various scenarios to be pointed out.

The Milstein method

The Milstein method for 1-dimensional case d=m=1 is given by:

$$F_{n+1} = F_n + a\Delta + b\Delta W + \frac{1}{2}bb'\{(\Delta w)^2 - \Delta\} \quad (11)$$

$$\Rightarrow F_{n+1} = F_n + \mathrm{a}\Delta + b\Delta W + \frac{1}{2}W\{\Delta w)^2$$

where a = a – ½ bb′ and b = b (Γ_n, F_n)

Taylor's approximation method

The 1-dimensional case d=m=1 of the implicit order 2.0 strong Taylor scheme has Stratonovich form of:

$$F_{n+1} = F_n + \frac{1}{2}\{\underline{a}(F_{n+1}) + \underline{a}\}\Delta + b\Delta W + \underline{a}b^1\{\Delta W\Delta - \Delta Z\} + \underline{a}b'\left\{\Delta Z - \frac{1}{2}\Delta W\Delta\right\}$$
$$+\frac{1}{2}bb'(\Delta W)^2 + \frac{1}{3!}b(bb')'(\nabla W)^3 + \frac{1}{4!}b(b(bb')')'(\Delta W)^4 + \underline{a}(bb')'\mathcal{J}(0,1,1) +$$
$$b(\underline{a}b')'\mathcal{J}(1,0,1) + \mathrm{b}(a'b')\{\mathcal{J}_{(1,1,0)} -$$
$$\frac{1}{4}\ (\Delta W)^2\Delta\}$$

Runge-Kutta approximation method

We shall limit ourselves to additive noise and to the Stratonovich approach of order 2.0. This in 1-dimensional case d = m = 1 with implicit strong Rungekutta scheme order 2.0 is given by

$$F_{n+1} = F_n + \{\mathrm{a}(\tilde{N}_+) + \underline{\mathrm{a}}\,(\tilde{N}_-) - \frac{1}{2}(\underline{\mathrm{a}}\,(F_{n+1}) + \mathrm{a})\}\Delta + b\Delta W$$

where $\tilde{N}_{\pm} = F_n + \frac{1}{2}\mathrm{a}\Delta + \frac{1}{\Delta}b(\Delta\hat{Z} \pm \xi)$, $\Delta\hat{z} = \frac{1}{2}\Delta Z + \frac{1}{4}\Delta W\Delta$ and

$$\xi = \sqrt{J(1,1,0)\Delta - \frac{1}{2}(\Delta Z)^2 + \frac{1}{8}(\Delta W)^2 + \frac{1}{2}\mathrm{Z}\Delta Z\Delta^{-1} - \Delta W)^2\Delta^2}$$

Regime switching process

We discuss the regime switching process in relation to its command, along with asset pricing within regimes and exchange rates as often encountered in the stock exchange market.

Regime switching process

The regime switching process command creates a regime-switching process with the specified regimes and transition probability. Each of the regimes must be a one-dimensional stochastic process. The parameter S defines all possible regimes that are one-dimensional stochastic processes. Moves between different regimes are assumed to be governed by the $d \times d$ transition i probability matrix, P, with generic element $P_{j,k}$ defined as the probability of moving from regime k to regime. The parameter n is the number of regimes per year. This process can only be simulated with time steps per year, where m is a multiple of n. Assume for example that is a finite state Markov chain with regimes per year. If we simulate the process on the interval (0, 2) with 12 time steps, then the regime change can occur only at steps , , , , and.

Asset pricing with regimes

It is paramount to note that a regime switching model is not just a practical model that can match closely with financial returns of some statistical features. But at equilibrium specification, regimes can generate interesting and real dynamics in relations with risk- return higher dynamics moment and has ability to use various elements, non- linear series time patterns particularly. This is analyzed when regime at equilibrium level are considered .The proper illustration of asset model pricing using an agent with utility U consumption over time, and also a discount factor subjection γ. According to Lucas (1978), suppose L_t is the period and equity price pays dividend out is B_t.

$$L_t u^1(A_t) = \gamma E_t \quad (U^1(A_{t+1})(L_{t+1} + B_{t+1}) \qquad (13)$$

where E_t represents conditional expectation we let utility power

$$U(A) = A^{1+} \tau / \quad \tau \quad \tau \; / \quad (1+), \; = -1.$$

Also we can equally assume that the rate of consumption has equality with dividend at each period, $A_t = B_t$. Hence, the Euler Equation 1 implies

$$L_t B_t = \gamma \; E_t \quad B_{t+1}(L_{t+1} + B_{t+1}) \qquad (14)$$

The quantity $N_{t+1} = \beta U^1(A_{t+1}) / U^1(A_t)$ is termed stochastic to Lam and Mark (1990), the dividend process answer to switches in the mean and volatility:

$$B_{t+1} = B_t \frac{\omega \quad \omega}{\omega \quad \rho} \exp\{\overset{\rho}{_0} + \overset{\rho}{_1} S_{t+1} + (_0 + {_1} S_{t+1}) \varepsilon_{t+1}\} \quad \overset{\varepsilon \sim}{_t} \text{ iid } (0,1)$$

(15)

where S_t belongs to the interval (0, 1) o and o represent dividend growth volatility of the regime. This model's dividend or consumption growth assume strong support by empirical analyses (Bekaert and Liu, 2004).

Exchange rates

Exchange rate is persistently high in regime switching models. This has link with some monetary policy especially in some currencies Dahlquist and Gray (2000), such switch occurs from a float free regime to rate peg exchange or target zone. The investment in a very high rate currencies by borrowing currencies with low interest rates is known as exhibit periods of continuous profits having sudden high volatility (Ichiue and Koyama, 2007). The behaviour of regime switching of going up stairs and coming down by the lifter results from the monetary policy action shown by Plantin and Shin (2006).

Effects of volatility and drift parameters through the simulation of the (RKM, T & M) methods

Finance regime switching process

Regime switching process has the following parameters (P, S, I, N, T) where P is transition matrix, S is the vector, I represents initial state, N represents number of state per year and T is the time variable. We shall consider a regime switching process with 2 regimes. In the first regime, the process is a Brownian motion with zero drift and high volatility; in the second regime, the process behaves like a Brownian motion with high drift and low volatility. The transition probabilities are: 0.5 for moving to the second regime given that the process is in the first regime and for moving to the first regime given that the process is in the second regime. The process will have regimes per year, which means that the regimes can switch only at t=0.5, t=1.0 and 0.2

P := <<0.5, 0.5>|<0.2, 0.8>>;

$$P := \begin{bmatrix} 0.5 & 0.2 \\ 0.5 & 0.8 \end{bmatrix}$$

> S := [Brownian Motion(0, 0, 2.0), Brownian Motion(0, 0.5, 0.001)]:

> X := Regime Switching Process(P, S, 1, 2):
> Path Plot (X(t), t = 0..2, timesteps = 20, replications = 10, gridlines = true, thickness = 2, axes = BOXED, color = red. blue) (Figure 1).

> P= <<0.5, 0.5>|<0.2, 0.8>>;

$$P := \begin{bmatrix} 0.5 & 0.2 \\ 0.5 & 0.8 \end{bmatrix}$$

> S:= [Brownian Motion(0, 0, 2.0), 0.5*t];

$$S := [\_X2, 0.5\,t]$$

> X := Regime Switching Process(P, S, 1, 2, t);

$$X := \_X3$$

> Path Plot(X(t), t = 0..2, time steps = 20, replications = 10, thickness = 2, axes = BOXED, gridlines = true, color = red.blue) (Figure 2).

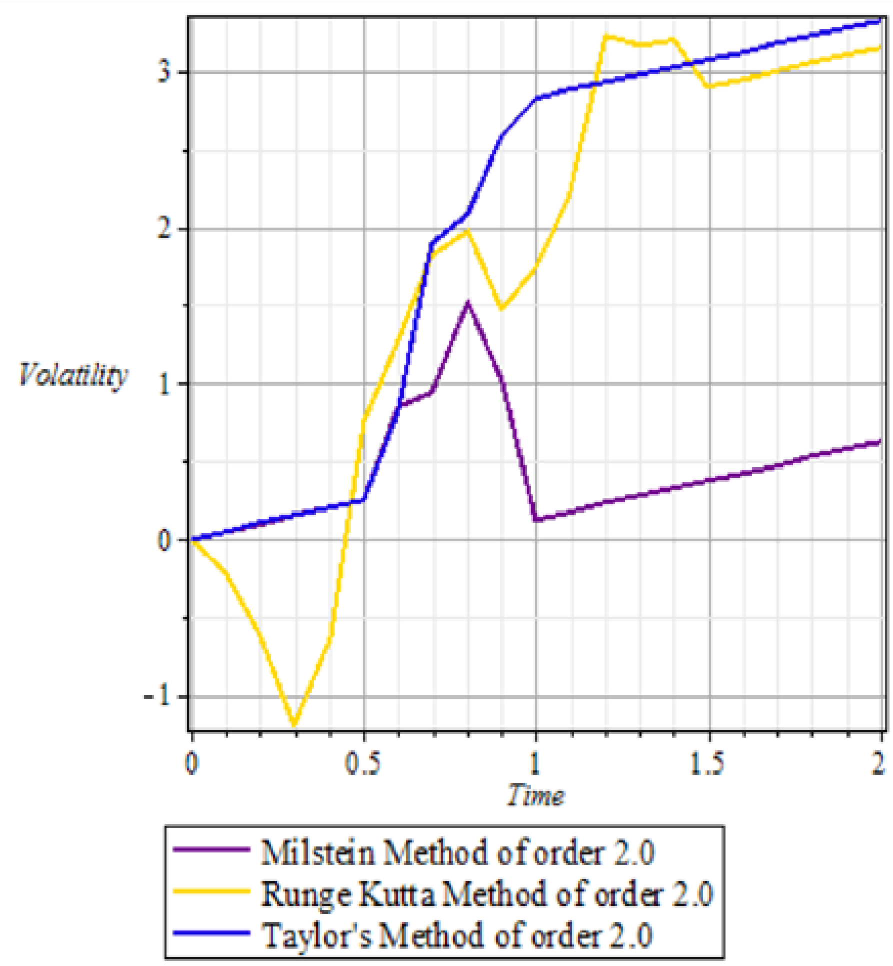

Figure 1. Regime switching process with zero drift and high volatility.

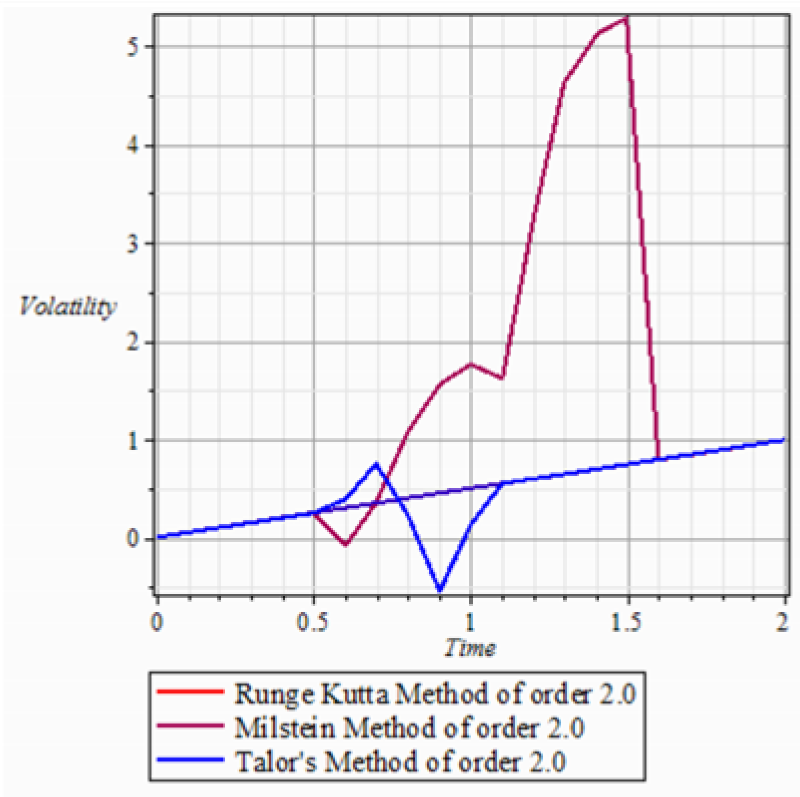

Figure 2. Regime switching process with high drift and low volatility.

Conclusion

Different approaches of solving Stochastic Differential Equations with certain numerical procedures only work in deterministic condition. However, the application of noise in differential equations makes it a non-deterministic for deterministic solvers. Also, the time discretization process considering Taylor's, Runge-Kutta and Milstein's methods in this research work has some numerical applications in several areas of discipline which this work intends to achieve.

The deterministic solvers of stochastic differential equations preferred Milstein, Taylor's and Runge-Kutta methods to other methods which include the explicit and implicit Euler, Nicolson Cranck scheme for the fact that the executions of the methods are much simpler. To execute Regime Switching Process with the aid of MAPLE 18 software, we assigned numerical values to different parameters. Hence, observed that Runge-Kutta method showed more rapid transition followed by Milstein and Taylor's methods when the drift is zero and the volatility is very high but when the drift is high and volatility is low Milstein method transit more rapidly followed by Runge-Kutta and Taylor's methods. Therefore, in order to checkmate stock business failure or progress we must apply a suitable method like Runge-Kutta as the better solver of instability problems. The following are the results obtained from the graphical presentations of the RungeKutta, Taylor and Milstein Methods in Figures 1 and 2.

i). The results of simulation approximation have shown that Milstein Method on Regime Switching Process is preferable followed by Runge-Kutta and Taylor's Methods.

ii). Rungekutta Method has shown more transition followed by Milstein and Taylor's Methods in the case of zero drift and high volatility while Milstein Method showed rapid transition followed by Runge-Kutta and Taylor's Methods in the case of high drift and low volatility.

These findings have shown that Runge-Kutta Method (RKM) is a better solver of stock related problems followed by Milstein and Taylor's Methods if the Stochastic process is very volatile with zero drift. Also, that Milstein Method is a better Stochastic approximation of the Regime Switching Process with high drift and low volatility.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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