ON APPROXIMATE SOLUTIONS TO A COUPLED ONE-DIMENSIONAL TIME FRACTIONAL KELLER-SEGEL MODEL

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The Daftardar-Jafari method (DJM) given by Daftardar-Gejji and Jafari is used to obtain approximate solutions to the time-fractional attraction Keller-Segel (TF-AKS) model in this work. The implementation of the method on the TF-AKS model is in two folds with respect to the chemotactic sensitivity function $\chi(v)$, namely: $\chi(v) = 1$and $\chi(v) = v$. The method consists of a very simple algorithm which is used to generate iterative solutions of the model. The result obtained further demonstrates the efficiency and reliability of the method, hence giving it a wider applicability to time-fractional order partial differential equations from mathematical biology.

Key words: Daftardar-Jafari method, time-fractional Keller-Segel model, Caputo derivative, approximate solution.

INTRODUCTION

Elements of fractional calculus have been extensively used in the formulation of linear and nonlinear mathematical models describing physical situations arising in dynamical control theory, electrochemistry, electrical circuits, feedback systems, biology, fluid and gas dynamics (Baleanu et al., 2012; Miller and Ross;1993; Oldham and Spanier, 2008; Podlubny, 1999; Samko et al., 1993). Determining exact analytic solutions to this class of problems proves more difficult than their classical integer-order counterparts. Fortunately, several approximation techniques for constructing analytic solutions for this class of problems have been developed and extensively used by many authors. Some of these techniques include the Adomian decomposition method (Dhaigude and Birajdar, 2012), variational iteration method, homotopy analysis method (Dehghan and Mana, 2010; Hashin et al., 2009), homotopy perturbation method (Momani and Odibat, 2007), homotopy decomposition method (Atangana and Alabaraoye; 2013), modified homotopy analysis transform method (Sunil et al., 2017) and differential transform method (Arikoglu and Ozkol, 2007; Odibat et al., 2008). Recently, Daftarder-Gejji and Jafari (2006) used the Daftarder-Jafari method (DJM) to construct approximate solutions to both linear and nonlinear differential equations with integer order or fractional order derivatives. This work applies the DJM to construct approximate analytic solutions to the one-dimensional time-fractional attraction Keller-Segel (TF-AKS) chemotaxis model:

$$\frac{\partial u(x,t)}{\partial t} = d_1 \frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\partial}{\partial x}\left(u(x,t)\frac{\partial v(x,t)}{\partial x}\right)$$
$$\frac{\partial v(x,t)}{\partial t} = d_2 \frac{\partial^2 v(x,t)}{\partial x^2} - \lambda v(x,t) + a u(x,t), \qquad (1)$$

with associated boundary conditions

$$\frac{\partial u(\alpha,t)}{\partial x} = \frac{\partial u(\beta,t)}{\partial x} = \frac{\partial v(\alpha,t)}{\partial x} = \frac{\partial v(\beta,t)}{\partial x} = 0, \quad (\alpha,\beta) \in I \qquad (2)$$

and initial conditions

$$u(x,0) = u_0(x), \quad v(x,0) = v_0(x) \qquad (3)$$

where $I \subset \mathbb{R}$ is a bounded open interval, d_1,

d_2, a, and λ are positive constants of biological importance, $\chi(v)$ is the chemotatic sensitivity function and and α is the parameter representing the order of the fractional derivative. The unknown functions, $u = u(x,t)$ and $v = v(x,t)$, denote the density of cellular species and concentration of the chemo-attractive substance, respectively. Additionally, the chemotatic term $D_x(u(x,t)D_x\chi(v(x,t)))$ measures the cellular sensitivity to the chemical. When $\alpha = 1$ and $\chi(v) = \chi v$ with $\chi > 0$, the system (1.1)-(1.3) reduces to the well-known classical one-dimensional attraction Keller-Segel chemotaxis model proposed in the 1970's by Keller and Segel (1970) to describe the aggregation process of cellular slime mold in response to an attractive chemical signal.

Chemotaxis is an essential means by which cellular entities interact within their environment. It is known to be a means of communication among motile marine organisms as they orient their motion either in the direction of an attraction-type chemical signal or away from a repulsion-type chemical signal in their quest for mates, nutrients and survival. It also dictates the process of self-organization and accounts for pattern formation in many biological species. Among higher organisms, chemotaxis plays a key role in cellular organization and positioning during embryogenesis, tumor cell invasion and cancer metastisis of living tissues (Hillen and Painter, 2009; Horstmann, 2003) and the references therein for detailed survey on the Keller-Segel chemotaxis model and several of its possible variants which have been studied from different mathematical perspectives.

Some important tools from fractional calculus

Some definitions and properties of fractional order differential operators from fractional calculus (Podlubny, 1999; Samko et al., 1993) are:

Definition 1

A real function $f(t), t > 0$ is said to be in the space $C_\mu, \mu \in \mathbb{R}$, if there exists a real number $p(> \mu)$ such that $f(t) = t^p g(t)$, where $g(t) \in C[0,\infty)$. It is said to be in the space C_μ^m *if* $f^{(m)} \in C_\mu, m \in \mathbb{N}$.

Definition 2

The Riemann-Liouville fractional integral of order $\alpha \geq 0$, of a function $f \in C_\mu, \mu \geq -1$, is defined as

$$I_t^\alpha f(t) = \begin{cases} f(t), & \alpha = 0, t > 0, \\ \frac{1}{\Gamma(\alpha)}\int_0^x (t-\eta)^{\alpha-1} f(\eta)d\eta, & \alpha > 0, t > 0. \end{cases} \quad (4)$$

Definition 3

The (left-sided) Caputo fractional derivative of order α of a function $f(t) \in C_{-1}^m$, is defined as

$$_{C}D_t^\alpha f(x,t) = I_t^{m-\alpha}\frac{\partial^m f(x,t)}{\partial t^m} = \begin{cases} \frac{1}{\Gamma(m-\alpha)}\left[\int_0^t (t-\eta)^{m-\alpha-1}\frac{d^m f(x,\eta)}{dt^m}d\eta\right], & m-1 < \alpha \leq m, m \in \mathbb{N}, \\ \frac{d^m f(x,t)}{dt^m}, & \alpha = m \in \mathbb{N}. \end{cases} \quad (5)$$

Note that

$$I_t^\alpha\left(cD_t^\alpha f(x,t)\right) = f(t) - \sum_{k=0}^{m-1} D^k f(x,0^+)\frac{t^k}{\Gamma(k+1)} \quad \text{and} \quad I_t^\alpha t^\gamma = \frac{\Gamma(\gamma+1)}{\Gamma(\gamma+\alpha+1)}t^{\gamma+\alpha}$$

FUNDAMENTAL IDEA OF THE DJM

The study considers the following nonlinear functional equation

$$\psi(x,t) = f(x,t) + N(\psi(x,t)). \quad (6)$$

Here, $\psi = \psi(x,t)$ is an unknown function,

$N: B \to B$ is a nonlinear operator on a Banach space B and $f = f(x,t)$ is a non-homogenous source term. The method asserts that the solution of (6) in the form of the indefinite series

$$\psi(x,t) = \sum_{k=0}^{\infty} \psi_k(x,t) \tag{7}$$

and the nonlinear operator N is expressed as the decomposition series:

$$N\left(\sum_{k=0}^{\infty} \psi_k\right) = N(\psi_0) + \sum_{k=1}^{\infty}\left[N\left(\sum_{j=0}^{k} \psi_j\right) - N\left(\sum_{j=0}^{k-1} \psi_j\right)\right] \tag{8}$$

Inserting (7) and (8) into (6), the study obtains

$$\sum_{k=0}^{\infty} \psi_k = f + N(\psi_k) + \sum_{k=1}^{\infty}\left[N\left(\sum_{j=0}^{k} \psi_j\right) - N\left(\sum_{j=0}^{k-1} \psi_j\right)\right] \tag{9}$$

from which the following recurrence relation was obtain

$$\begin{cases} \psi_0 = f \\ \psi_1 = N(\psi_o) \\ \psi_{n+1} = N\left(\sum_{k=0}^{n} \psi_k\right) - N\left(\sum_{k=0}^{n-1} \psi_k\right), \qquad n = 1,2,\cdots \end{cases} \tag{10}$$

It is easy to see from (10) that

$$(\psi_1 + \psi_2 + \cdots + \psi_{n+1}) = N(\psi_0 + \psi_1 + \psi_2 + \cdots + \psi_n), \quad n = 1,2,\cdots, \tag{11}$$

and

$$\sum_{k=0}^{\infty} \psi_k(x,t) = f(x,t) + N\left(\sum_{k=0}^{\infty} \psi_k(x,t)\right). \tag{12}$$

The *nth*-term approximation of the solution of (6) and (7) is then given by the truncated series $\sum_{k=0}^{n-1} \psi_k(x,t)$ ·
Now, if N is a contraction mapping in B, that is,

$$\|N(x) - N(y)\| \le k\|x - y\|, \qquad 0 < k < 1,$$

then

$$\|\psi_{n+1}\| = \left|\left|N(\psi_0 + \psi_1 + \psi_2 + \cdots + \psi_n) - N(\psi_0 + \psi_1 + \psi_2 + \cdots + \psi_{n-1})\right|\right| \le k^{n+1}\|\psi_0\|, \quad n = 0,1,2,\cdots.$$

and the series $\sum_{k=0}^{\infty} \psi_k(x)$ converges absolutely and uniformly to a solution of (6) (Cherruault, 1989) and is unique owing to the Banach fixed point theorem (Jerri, 1999). The convergence of the DJM has been proved in Hemeda (2013) and Bhalekar and Daftardar-Gejji (2011).

DJM ALGORITHM FOR COUPLED SYSTEM OF TIME-FRACTIONAL PDES

Consider the following nonlinear coupled system of time-fractional PDEs

$$\begin{aligned} D_t^{\alpha} u_1 &= A_1(u,v,\partial u,\partial v) + B_1(x,t), \\ D_t^{\alpha} u_2 &= A_2(u,v,\partial u,\partial v) + B_2(x,t), \end{aligned} \tag{13}$$

with initial data

$$\frac{\partial^k}{\partial t^k} u(x,0) = h_k^1(x), \quad \frac{\partial^k}{\partial t^k} v(x,0) = h_k^2(x), \quad k = 0,1,2,\cdots, m-1, \tag{14}$$

where $m - 1 < \alpha \le m \in \mathbb{N}$, A_1 and A_2 are nonlinear functions of u_i and their partial derivatives and B_1 and B_2 are inhomogeneous source terms. Taking appropriate fractional integral operator I_t^{α} from 0 to t on both sides of each equation in (13), it was easy to transform (14) to a system of nonlinear functional equations:

$$\begin{aligned} u_1(x,t) &= f_1 + N_1(u,v), \\ u_2(x,t) &= f_2 + N_2(u,v), \end{aligned} \tag{15}$$

Where

$$\begin{cases} f_i = \sum_{k=0}^{m-1} \frac{t^k}{\Gamma(k+1)} h_k^i(x) + I_t^{\alpha} B_i(x,t), \\ N_i(u,v) = I_t^{\alpha} A_i(u,v,\partial u,\partial v). \end{cases} \tag{16}$$

For $i = 1,2,$ and (7)-(10) was obtained from the recursive relations

$$\begin{cases} u_0(x,t) = f_1(x,t), \\ v_0(x,t) = f_2(x,t)\,, \\ u_1(x,t) = N_1\big(u(x,t), v(x,t)\big)\,, \\ v_1(x,t) = N_2\big(u(x,t), v(x,t)\big), \\ u_{m+1}(x,t) = N_1\left(\sum_{k=0}^{m} u_k(x,t)\,, \sum_{k=0}^{m} v_k(x,t)\right) - N_1\left(\sum_{k=0}^{m-1} u_k(x,t)\,, \sum_{k=0}^{m-1} v_k(x,t)\right), \\ v_{m+1}(x,t) = N_2\left(\sum_{k=0}^{m-1} u_k(x,t)\,, \sum_{k=0}^{m-1} v_k(x,t)\right) - N_2\left(\sum_{k=0}^{m-1} u_k(x,t)\,, \sum_{k=0}^{m-1} v_k(x,t)\right), \\ m = 1,2,\ldots, \end{cases} \tag{17}$$

from which the entire solution components of the IVP (13)-(14) computed.

Approximate solution of the TF-AKS model using the DJM

The DJM was used to obtain approximate analytic solution to the TF-AKS model (1)-(3). To this end, two cases were consider with respect to the chemotatic sensitivity function, namely, $\chi(v) = 1$ and $\chi(v) = v$ subject to the initial conditions

$$u(x,0) = k_1 e^{-x^2}, \quad v(x,0) = k_2 e^{-x^2}. \tag{18}$$

Example 1

Assume that $\chi(v) = 1$, then the TF-AKS model (1) reads

$$\begin{cases} \frac{\partial u(x,t)}{\partial t} = d_1 \frac{\partial^2 u(x,t)}{\partial x^2}, \\ \frac{\partial v(x,t)}{\partial t} = d_2 \frac{\partial^2 v(x,t)}{\partial x^2} - \lambda v(x,t) + a u(x,t). \end{cases} \tag{19}$$

Operating both sides of each equation in (19) by I_t^{α} and keeping track of the initial conditions in (18), also obtained was the following equivalent system of fractional integral equations

$$u(x,t) = f_1 + N_1(u,v), \quad v(x,t) = f_2 + N_2(u,v) \tag{20}$$

where $f_1 = k_1 e^{-x^2}, f_2 = k_2 e^{-x^2}$ and the nonlinear terms $N_1(u,v)$, $N_2(u,v)$ are defined as

$$\begin{cases} N_1(u,v) = I_t^{\alpha}\left[d_1 \frac{\partial^2 u(x,t)}{\partial x^2}\right], \\ N_2(u,v) = I_t^{\alpha}\left[d_2 \frac{\partial^2 v(x,t)}{\partial x^2} - \lambda v(x,t) + a u(x,t)\right] \end{cases} \tag{21}$$

Accordingly, the solutions for (19) is giving by the series

$$\begin{cases} u(x,t) = \sum_{k=0}^{\infty} u_k(x,t) \\ v(x,t) = \sum_{k=0}^{\infty} v_k(x,t). \end{cases}$$

Furthermore, by the same steps leading to the recurrence relation (17), the following iterates were obtained:

$$u_0(x,t) := k_1 e^{-x^2},$$

$$v_0(x,t) := k_2 e^{-x^2},$$

$$u_1(x,t) := \frac{2d_1k_1(2x^2-1)e^{-x^2}t^{\alpha}}{\Gamma(\alpha+1)},$$

$$v_1(x,t) := \frac{(4x^2d_2k_2 + ak_1 - \lambda k_2 - 2d_2k_2)e^{-x^2}t^{\alpha}}{\Gamma(\alpha+1)},$$

$$u_2(x,t) := \frac{4d_1^2k_1(4x^4-12x^2+3)e^{-x^2}t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{2d_1k_1(2x^2-1)e^{-x^2}t^{\alpha}}{\Gamma(\alpha+1)}$$

$$v_2(x,t) := \frac{\big(4d_2^2k_2(4x^4-12x^2+3) + 2(2x^2-1)(ad_1k_1 + ad_2k_1 - 2\lambda d_2k_2) - \lambda(ak_1 - \lambda k_2)\big)e^{-x^2}t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{(4x^2d_2k_2 + ak_1 - \lambda k_2 - 2d_2k_2)e^{-x^2}t^{\alpha}}{\Gamma(\alpha+1)},$$

$$u_3(x,t) := \frac{8d_1^3k_1(8x^6-60x^4+90x^2-15)e^{-x^2}t^{3\alpha}}{\Gamma(3\alpha+1)} - \frac{8d_1^2k_1(4x^4-12x^2+3)e^{-x^2}t^{2\alpha}}{\Gamma(2\alpha+1)},$$

$$v_3(x,t) := \frac{8d_2^3k_2(8x^6-60x^4+90x^2-15)e^{-x^2}t^{3\alpha}}{\Gamma(3\alpha+1)} - \frac{2\lambda(2x^2-1)(ad_1k_1 + 2ad_2k_1 - 3\lambda d_2k_2)e^{-x^2}t^{3\alpha}}{\Gamma(3\alpha+1)} + \frac{4(4x^4-12x^2+3)(ad_1^2k_1 + ad_1d_2k_1 + ad_2^2k_1 - 3\lambda d_2^2k_2)e^{-x^2}t^{3\alpha}}{\Gamma(3\alpha+1)} + \frac{\lambda^2(ak_1-\lambda k_2)e^{-x^2}t^{3\alpha}}{\Gamma(3\alpha+1)} - \frac{8d_2^2k_2(4x^4-12x^2+3)e^{-x^2}t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{2(4x^2-2)(ad_1k_1 + ad_2k_1 - 2\lambda d_2k_2)e^{-x^2}t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{2\lambda(ak_1-\lambda k_2)e^{-x^2}t^{2\alpha}}{\Gamma(2\alpha+1)}.$$

For a better approximation, the study continues in a similar manner to obtain more solution components for $u_m(x,t)$ and $v_m(x,t)$ for $m \geq 4$. Finally the solution of the system is given as:

$$\begin{aligned} u(x,t) &= u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + \cdots \\ v(x,t) &= v_0(x,t) + v_1(x,t) + v_2(x,t) + v_3(x,t) + \cdots \end{aligned} \tag{22}$$

Example 2

Assume that $\chi(v) = v$, then the TF-AKS model (1) reads

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} = d_1\dfrac{\partial^2 u(x,t)}{\partial x^2} - \dfrac{\partial u(x,t)}{\partial x}\dfrac{\partial v(x,t)}{\partial x} - u(x,t)\dfrac{\partial^2 v(x,t)}{\partial x^2}, \\ \dfrac{\partial v(x,t)}{\partial t} = d_2\dfrac{\partial^2 v(x,t)}{\partial x^2} - \lambda v(x,t) + au(x,t). \end{cases} \tag{23}$$

Operating both sides of each equation in (23) by I_t^{α} and keeping track of the initial conditions in (18), the following equivalent system of fractional integral equations were obtained

$$\begin{cases} u(x,t) = f_1 + N_1(u,v), \\ v(x,t) = f_2 + N_2(u,v), \end{cases} \tag{24}$$

where $f_1 = k_1 e^{-x^2}, f_2 = k_2 e^{-x^2}$ and the nonlinear terms $N_1(u,v)$, $N_2(u,v)$ are defined as

$$\begin{cases} N_1(u,v) = I_t^{\alpha}\left[d_1 \frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\partial u(x,t)}{\partial x}\frac{\partial v(x,t)}{\partial x} - u(x,t)\frac{\partial^2 v(x,t)}{\partial x^2}\right] \\ N_2(u,v) = I_t^{\alpha}\left[d_2 \frac{\partial^2 v(x,t)}{\partial x^2} - \lambda v(x,t) + a u(x,t)\right]. \end{cases} \tag{25}$$

Accordingly, the solutions for (23) is given by the series

$$\begin{cases} u(x,t) = \sum_{k=0}^{\infty} u_k(x,t), \\ v(x,t) = \sum_{k=0}^{\infty} v_k(x,t). \end{cases}$$

Furthermore, by the same steps leading to the recurrence relation (17), the following few iterates were obtained:

$$u_0(x,t) := k_1 e^{-x^2},$$
$$v_0(x,t) := k_2 e^{-x^2},$$
$$u_1(x,t) := -\frac{2e^{-x^2}k_1\left(4k_2x^2e^{-x^2} - 2x^2d_1 - k_2e^{-x^2} + d_1\right)t^\alpha}{\Gamma(\alpha+1)},$$
$$v_1(x,t) := \frac{e^{-x^2}(4x^2d_2k_2 + ak_1 - \lambda k_2 - 2d_2k_2)t^\alpha}{\Gamma(\alpha+1)},$$

$$u_2(x,t)$$
$$:= \frac{1}{\Gamma(3\alpha+1)\Gamma(\alpha+1)^2}\left(8x^2e^{-2x^2}k_1\left(8k_2x^2e^{-x^2} - 2x^2d_1 - 6k_2e^{-x^2} + 3d_1\right)(4x^2d_2k_2 + ak_1 - \lambda k_2 - 6d_2k_2)\Gamma(2\alpha+1)t^{3\alpha}\right)$$
$$+ \frac{1}{\Gamma(3\alpha+1)\Gamma(\alpha+1)^2}\left(4e^{-2x^2}k_1\left(4k_2x^2e^{-x^2} - 2x^2d_1 - k_2e^{-x^2} + d_1\right)(8x^4d_2k_2 + 2ax^2k_1 - 2\lambda x^2k_2 - 24x^2d_2k_2 - ak_1 + \lambda k_2 + 6d_2k_2)\Gamma(2\alpha+1)t^{3\alpha}\right)$$
$$+ \frac{2e^{-x^2}k_1\left(4k_2x^2e^{-x^2} - 2x^2d_1 - k_2e^{-x^2} + d_1\right)\left(-2k_2e^{-x^2} + 4k_2x^2e^{-x^2}\right)t^{2\alpha}}{\Gamma(2\alpha+1)}$$
$$- \frac{4d_1e^{-x^2}k_1\left(32k_2x^4e^{-x^2} - 4x^4d_1 - 48k_2x^2e^{-x^2} + 12x^2d_1 + 6k_2e^{-x^2} - 3d_1\right)t^{2\alpha}}{\Gamma(2\alpha+1)}$$
$$+ \frac{2e^{-x^2}k_1\left(4k_2x^2e^{-x^2} - 2x^2d_1 - k_2e^{-x^2} + d_1\right)t^\alpha}{\Gamma(\alpha+1)}$$
$$+ \frac{8x^2e^{-2x^2}k_1\left(8k_2x^2e^{-x^2} - 2x^2d_1 - 6k_2e^{-x^2} + 3d_1\right)k_2t^{2\alpha}}{\Gamma(2\alpha+1)},$$

$$v_2(x,t) := \frac{4d_2^2k_2(4x^4 - 12x^2 + 3)e^{-x^2}t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{2\left(e^{-x^2}\right)^2ak_1k_2(2x-1)(2x+1)t^{2\alpha}}{\Gamma(2\alpha+1)}$$
$$+ \frac{2(2x^2-1)(ad_1k_1 + ad_2k_1 - 2\lambda d_2k_2)e^{-x^2}t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{\lambda(ak_1 - \lambda k_2)e^{-x^2}t^{2\alpha}}{\Gamma(2\alpha+1)}$$
$$- \frac{e^{-x^2}(4x^2d_2k_2 + ak_1 - \lambda k_2 - 2d_2k_2)t^\alpha}{\Gamma(\alpha+1)},$$

For a better approximation, the study continues with the same iterative steps to obtain more solution components for $u_m(x,t)$ and $v_m(x,t)$ for $m \geq 3$. Finally the most approximate solution of the system is given as:

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + \cdots$$
$$v(x,t) = v_0(x,t) + v_1(x,t) + v_2(x,t) + \cdots$$
(26)

Numerical discussions

Discussions of the numerical results obtained for the TF-AKS models were provided in **Example 1** and **Example 2** above. The numerical simulations via graphical representations are given in 2D and 3D for the approximate solutions for distinct values of α and for the following set of theoretical parameters:

$d_1 = 0.5, \quad d_2 = 3, \quad k_1 = 120, \quad k_2 = 160, \quad a = 1, \quad \lambda = 2, \quad \chi = 1.$

Figures 1(a)-(d) shows the surface graphs for the chemotatic cell density and chamoattractant concentration in Example 1 different values of α while Figures 2(a)-(d) shows the surface graphs for the chemotatic cell density and chamoattractant concentration in Example 2 for different values of α. For the different values of α, the graphs show close similarities, indicating continuous dependence on the fractional parameter α.

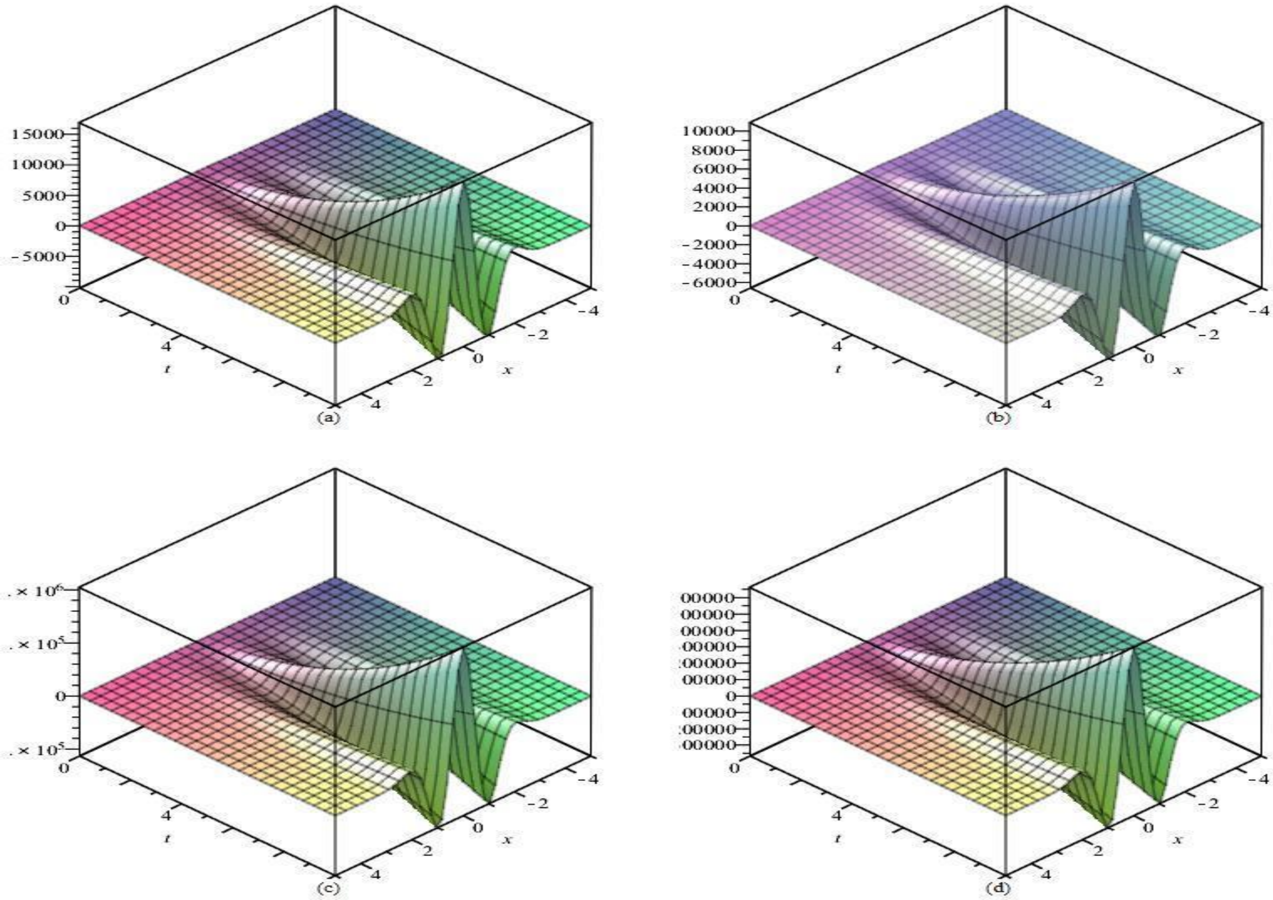

Figure 1. (a)-(d): 3D surface graphs representations for the approximate solution (22): (a) u(x, t) at α = 1 (b) u(x, t) at α = 0.85 (c) v(x, t) at α = 1 (d) v(x, t) at α = 0.85.

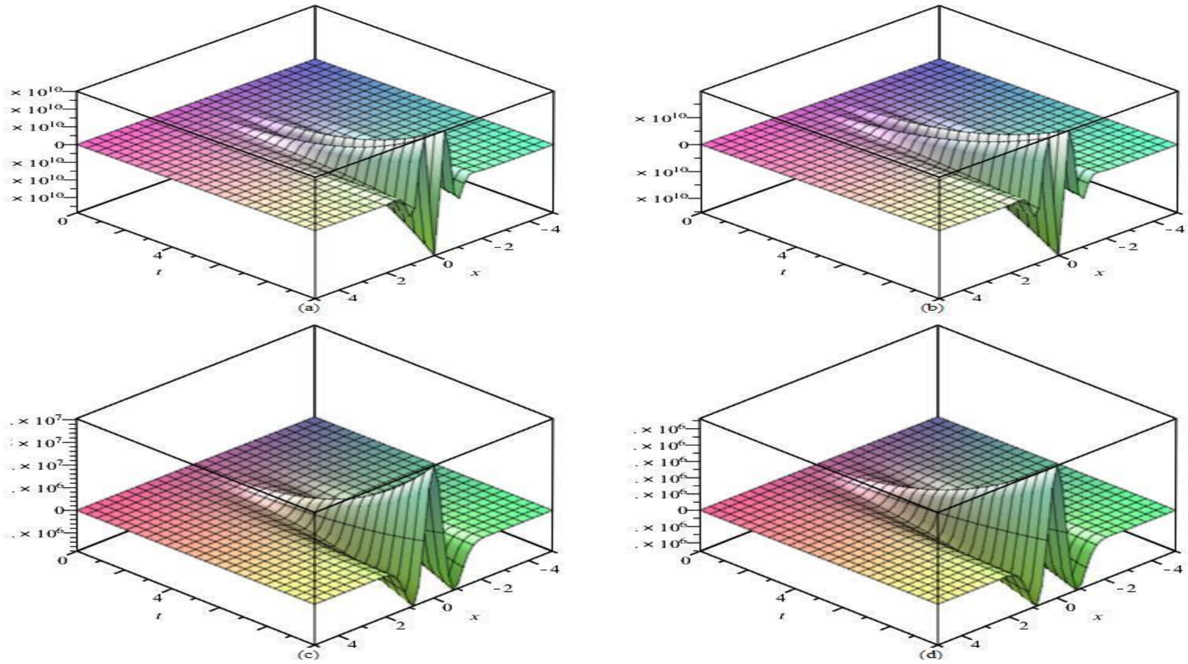

Figure 2. (a)-(d): 3D surface graphs representations for the approximate solution (26): (a) u(x, t) at α = 1 (b) u(x, t) at α = 0.85 (c) v(x, t) at α = 1 (d) v(x, t) at α = 0.85.

Conclusions

The mathematical model for the one-dimensional time-fractional attraction Keller-Segel (TF-AKS) model with different chemotactic sensitivity functions were investigated in this paper, using the DJM. The proposed method works very well for both linear and nonlinear systems of differential equations with either integer order or fractional order differential operators. One very important advantage of the technique used is that it does not require discretization of variables, perturbation or any form of restrictive assumptions. It is straightforward, very easy to implement and computationally attractive than many other methods. It is also a very efficient analytical method which provides a simple iterative algorithm that could be extended to a wide class of models described by coupled systems of linear and nonlinear time-fractional partial differential equations arising in mathematical biology and other areas of science.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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